MCSS 0601
A digital PI-controller can be defined as
\[ u(k) = u(k-1) + K \left[ (1 + \frac{h}{T}) y(k) - y(k-1) \right] \]
where \( K, h \) and \( T \) are positive real numbers. Which is the most general ARMAX system for which the PI controller above, with suitable choice of parameters, is identical to a minimum variance controller?

MCSS 0602
A signal can be described by the following autocovariance function:
\[ r(0) = 1.0, \ r(1) = 0.8, \ r(2) = 0.4, \ r(3) = 0.2, \ r(4) = -0.1, \ r(5) = 0.1, \ r(h) = 0, \ h > 5. \]
The signal will be filtered with a digital lowpass filter, which (at least approximately) blocks frequencies over \( \pi/2 \) and lets lower frequencies pass through. Determine the variance of the filtered signal.

MCSS 0603
Consider a system described by the difference equation
\[ y(k+1) = ay(k) + e(k+1) + ce(k) \]
where \( a = 0.8, \ c = 0.6 \) and \( \{e(k)\} \) is white noise with mean zero and variance \( \sigma_e^2 = 1.2 \). Measurements \( y(k), \ k = 0, \ldots, N, \) from the output of the system is gathered and the estimate \( \hat{a}_N \) of \( a \) is calculated using the least squares method.
Towards which value will the estimate \( \hat{a}_N \) converge when \( N \) grows towards infinity?

MCSS 0604
A system is described by the model
\[ y(k) - 1.5y(k-1) + 0.4y(k-2) = 0.8v(k-1) + e(k) \]
where \( \{e(k)\} \) is white noise with the variance 1 and \( v(k) \) is the output signal from another system, which can be described by the model
\[ v(k) - 0.9v(k-1) = e_v(k) + 0.3e_v(k-1) \]
where \( \{e_v(k)\} \) is white noise with the variance 1. The noise sequences \( \{e(k)\} \) and \( \{e_v(k)\} \) are independent from each other.
Assume that the signal \( \{v(k)\} \) can be measured without error. However, the measurement system delays the signal, so that the measurement \( v(k) \) is available at first at the time \( k+1 \). Determine the optimal 2-step predictor \( \hat{y}(k+2|k) \) which utilizes both the output signal \( y \) and this measurement of \( v \). What is the minimal variance for the prediction error?