Benchmarking and linguistic importance weighted aggregations

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Abstract

In this work we concentrate on the issue of weighted aggregations and provide a possibilistic approach to the process of importance weighted transformation when both the importances (interpreted as benchmarks) and the ratings are given by symmetric triangular fuzzy numbers. We will show that using the possibilistic approach (i) small changes in the membership function of the importances can cause only small variations in the weighted aggregate; (ii) the weighted aggregate of fuzzy ratings remains stable under small changes in the nonfuzzy importances; (iii) the weighted aggregate of crisp ratings still remains stable under small changes in the crisp importances whenever we use a continuous implication operator for the importance weighted transformation. We illustrate the proposed method by a simple example.

1 Introduction

In many applications of fuzzy sets such as multi-criteria decision making, pattern recognition, diagnosis and fuzzy logic control one faces the problem of weighted aggregation. The issue of weighted aggregation has been studied extensively by Carlsson and Fullér [1, 2, 3], Delgado et al [4, 5], Dubois and Prade [6, 7, 8], Fodor and Roubens [9] Herrera et al [12, 13, 14, 15, 16] and Yager [20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. Unlike Herrera and Herrera-Viedma [15] who perform direct computation on a finite and totally ordered term set, we use the membership functions to aggregate the values of the linguistic variables rate and importance. The main problem with finite term sets is that the impact of small changes in the weighting vector can be disproportionately large on the weighted aggregate (because the set of possible output values is finite, but the set of possible weight vectors is a subset of \( \mathbb{R}^n \)). For example, the rounding operator in the convex combination of linguistic labels, defined by Delgado et al. [4], is very sensitive to the values around 0.5 (\( \text{round}(0.499) = 0 \) and \( \text{round}(0.501) = 1 \)).

In this paper we consider the process of importance weighted aggregation when both the aggregates and the importances are given by an infinite term set, namely by the values of the linguistic variables "rate" and "importance". In our approach the importances are considered as benchmark levels for the performances, i.e. an alternative performs well on all criteria if the degree of satisfaction to each of the criteria is at least as big as the associated benchmark. The proposed "stable" method ranks the alternatives by measuring the degree to which they satisfy the proposition: "All ratings are larger than or equal to their importance". We will also use OWA operators to measure the degree to which an alternative satisfies the proposition: "Most ratings are larger than or equal to their importance", where the OWA weights are derived from a well-chosen linguistic quantifier.

### 2 Benchmarking and weighted aggregations

**Definition 2.1** A fuzzy set \( A \) is called a symmetric triangular fuzzy number with center \( a \) and width \( \alpha > 0 \) if its membership function has the following form

\[
A(t) = \begin{cases} 
1 - \frac{|a - t|}{\alpha} & \text{if } |a - t| \leq \alpha \\
0 & \text{otherwise}
\end{cases}
\]

and we use the notation \( A = (a, \alpha) \). If \( \alpha = 0 \) then \( A \) collapses to the characteristic function of \( \{a\} \subset \mathbb{R} \) and we will use the notation \( A = \bar{a} \).

We will use symmetric triangular fuzzy numbers to represent the values of linguistic variables [30] rate and importance in the universe of discourse \( I = [0, 1] \). The set of all symmetric triangular fuzzy numbers in the unit interval will be denoted by \( \mathcal{R}(I) \).

**Definition 2.2** Let \( A = (a, \alpha) \) and \( B = (b, \beta) \) The degree of possibility that the proposition ”\( A \) is less than or equal to \( B \)” is true, denoted by \( \text{Pos}[A \leq B] \), is defined as

\[
\text{Pos}[A \leq B] = \sup_{x \leq y} \min\{A(x), B(y)\},
\]

and computed by

\[
\text{Pos}[A \leq B] = \begin{cases} 
1 & \text{if } a \leq b \\
1 - \frac{a - b}{\alpha + \beta} & \text{if } 0 < a - b < \alpha + \beta \\
0 & \text{otherwise}
\end{cases}
\]

(1)

It is easy to see that

\[
\text{Pos}[A \leq B] = \sup_{x - y \leq 0} \min\{A(x), B(y)\} = \sup_{t \leq 0} (A - B)(t)
\]

(2)

The Hausdorff distance of \( A \) and \( B \), denoted by \( D(A, B) \), is defined by [11]

\[
D(A, B) = \max_{\theta \in I} \max \{|a_1(\theta) - b_1(\theta)|, |a_2(\theta) - b_2(\theta)|\}
\]

where \([a_1(\theta), a_2(\theta)]\) and \([b_1(\theta), b_2(\theta)]\) denote the \( \theta \)-level sets of \( A \) and \( B \), respectively.

Figure 1: \( \text{Pos}[A \leq B] < 1 \), because \( a > b \).
Lemma 2.1 [10] Let $\delta > 0$ be a real number and let $A = (a, \alpha), B = (b, \beta)$ be symmetric triangular fuzzy numbers. Then from the inequality
\[ D(A, B) \leq \delta \]
it follows that
\[ \sup_{t \in \mathbb{R}} |A(t) - B(t)| \leq \max\{1/\alpha, 1/\beta\} \delta \] (3)

Definition 2.3 $R$-implications are obtained by residuation of a continuous $t$-norm $T$, i.e.
\[ x \rightarrow y = \sup\{z \in [0,1] \mid T(x, z) \leq y\} \]
These implications arise from the Intuitionistic Logic formalism. We shall use the following $R$-implication:
\[ x \rightarrow y = \begin{cases} 1 & \text{if } x \leq y \\ y & \text{otherwise} \end{cases} \quad \text{(Gödel implication)} \] (4)
\[ x \rightarrow y = \min\{1 - x + y, 1\} \quad \text{(Łukasiewicz implication)} \] (5)

Let $A$ be an alternative with ratings $(A_1, A_2, \ldots, A_n)$, where $A_i = (a_i, \alpha_i) \in \mathcal{R}(I)$, $i = 1, \ldots, n$. For example, the symmetric triangular fuzzy number $A_j = (0.8, \alpha)$ when $0 < \alpha \leq 0.2$ can represent the property
"the rating on the $j$-th criterion is around 0.8"
and if $\alpha = 0$ then $A_j = (0.8, \alpha)$ is interpreted as
"the rating on the $j$-th criterion is equal to 0.8"
and finally, the value of $\alpha$ can not be bigger than 0.2 because the domain of $A_j$ is the unit interval.
Assume that associated with each criterion is a weight $W_i = (w_i, \gamma_i)$ indicating its importance in the aggregation procedure, $i = 1, \ldots, n$.
For example, the symmetric triangular fuzzy number $W_j = (0.5, \gamma) \in \mathcal{R}(I)$ when $0 < \gamma \leq 0.5$ can represent the property
"the importance of the $j$-th criterion is approximately 0.5"
and if $\gamma = 0$ then $W_j = (0.5, \gamma)$ is interpreted as
"the importance of the $j$-th criterion is equal to 0.5"
and finally, the value of $\gamma$ can not be bigger than 0.5 because the domain of $W_j$ is the unit interval.
The general process for the inclusion of importance in the aggregation involves the transformation of the ratings under the importance. In this paper we suggest the use of the transformation function
\[ g: \mathcal{R}(I) \times \mathcal{R}(I) \rightarrow [0,1], \]
where,
\[ g(W_i, A_i) = \text{Pos}[W_i \leq A_i], \]
for $i = 1, \ldots, n$, and then obtain the weighted aggregate,
\[ \phi(A, W) = \text{Agg}\{\text{Pos}[W_1 \leq A_1], \ldots, \text{Pos}[W_n \leq A_n]\}. \] (6)
where $\text{Agg}$ denotes an aggregation operator.
For example if we use the \textit{min} function for the aggregation in (6), that is,
\[
\phi(A, W) = \min\{\text{Pos}[W_1 \leq A_1], \ldots, \text{Pos}[W_n \leq A_n]\}
\]
then the equality
\[
\phi(A, W) = 1
\]
holds iff \( w_i \leq a_i \) for all \( i \), i.e. when the mean value of each performance rating is at least as large as the mean value of its associated weight. In other words, if a performance rating with respect to a criterion exceeds the importance of this criterion with possibility one, then this rating does not matter in the overall rating. However, ratings which are well below the corresponding importances (in possibilistic sense) play a significant role in the overall rating. In this sense the importance can be considered as \textit{benchmark} or \textit{reference level} for the performance. Thus, formula (6) with the \textit{min} operator can be seen as a measure of the degree to which an alternative satisfies the following proposition:

\"All ratings are larger than or equal to their importance\".

It should be noted that the \textit{min} aggregation operator does not allow any compensation, i.e. a higher degree of satisfaction of one of the criteria can not compensate for a lower degree of satisfaction of another criterion.

Averaging operators realize \textit{trade-offs} between criteria, by allowing a positive compensation between ratings. We can use an \textit{andlike} or an \textit{orlike} OWA-operator [23] to aggregate the elements of the bag
\[
\langle \text{Pos}[W_1 \leq A_1], \ldots, \text{Pos}[W_n \leq A_n] \rangle.
\]
In this case (6) becomes,
\[
\phi(A, W) = \text{OWA}(\text{Pos}[W_1 \leq A_1], \ldots, \text{Pos}[W_n \leq A_n]),
\]
where \textit{OWA} denotes an Ordered Weighted Averaging Operator introduced by Yager in [21].

\textbf{Remark 2.1} Formula (6) does not make any difference among alternatives whose performance ratings exceed the value of their importance with respect to all criteria with possibility one: the overall rating will always be equal to one. Penalizing ratings that are \"larger than the associated importance, but not large enough\" (that is, their intersection is not empty) we can modify formula (6) to measure the degree to which an alternative satisfies the following proposition:

\"All ratings are essentially larger than their importance\".

In this case the transformation function can be defined as
\[
g(W_i, A_i) = \text{Nes}[W_i \leq A_i] = 1 - \text{Pos}[W_i > A_i],
\]
for \( i = 1, \ldots, n \), and then obtain the weighted aggregate,
\[
\phi(A, W) = \min\{\text{Nes}[W_1 \leq A_1], \ldots, \text{Nes}[W_n \leq A_n]\}.
\]
(8)

If we do allow a positive compensation between ratings then we can use OWA-operators in (8). That is,
\[
\phi(A, W) = \text{OWA}(\text{Nes}[W_1 \leq A_1], \ldots, \text{Nes}[W_n \leq A_n]).
\]

The following theorem shows that if we choose the \textit{min} operator for \texttt{Agg} in (6) then small changes in the membership functions of the weights can cause only a small change in the weighted aggregate, i.e. the weighted aggregate depends continuously on the weights.
Theorem 2.1 Let $A_i = (a_i, \alpha) \in \mathcal{R}(I)$, $\alpha_i > 0$, $i = 1, \ldots, n$ and let $\delta > 0$ such that

$$\delta < \alpha := \min\{\alpha_1, \ldots, \alpha_n\}$$

If $W_i = (w_i, \gamma_i)$ and $W_i^\delta = (w_i^\delta, \gamma_i^\delta) \in \mathcal{R}(I)$, $i = 1, \ldots, n$, satisfy the relationship

$$\max_i D(W_i, W_i^\delta) \leq \delta$$

(9)

then the following inequality holds,

$$|\phi(A, W) - \phi(A, W^\delta)| \leq \frac{\delta}{\alpha}$$

(10)

where $\phi(A, W)$ is defined by (7) and

$$\phi(A, W^\delta) = \min\{\text{Pos}[W_1^\delta \leq A_1], \ldots, \text{Pos}[W_n^\delta \leq A_n]\}.$$

Proof. It is sufficient to show that

$$|\text{Pos}[W_i \leq A_i] - \text{Pos}[W_i^\delta \leq A_i]| \leq \frac{\delta}{\alpha}$$

(11)

for $1 \leq i \leq n$, because (10) follows from (11). Using the representation (2) we need to show that

$$|\sup_{t \leq 0}(W_i - A_i) - \sup_{t \leq 0}(W_i^\delta - A_i)| \leq \frac{\delta}{\alpha}.$$

Using (9) and applying Lemma 2.1 to

$$W_i - A_i = (w_i - a_i, \alpha_i + \gamma_i) \text{ and } W_i^\delta - A_i = (w_i - a_i, \alpha_i + \gamma_i^\delta),$$

we find

$$D(W_i - A_i, W_i^\delta - A_i) = D(W_i, W_i^\delta) \leq \delta,$$

and

$$\sup_{t \leq 0}|(W_i - A_i)(t) - (W_i^\delta - A_i)(t)| \leq \sup_{t \leq 0}|(W_i - A_i)(t) - (W_i^\delta - A_i)(t)| \leq \max\left\{\frac{1}{\alpha_i + \gamma_i}, \frac{1}{\alpha_i + \gamma_i^\delta}\right\} \times \delta \leq \frac{\delta}{\alpha}.$$

Which ends the proof.

Remark 2.2 From (9) and (10) it follows that

$$\lim_{\delta \to 0} \phi(A, W^\delta) = \phi(A, W)$$

for any $A$, which means that if $\delta$ is small enough then $\phi(A, W^\delta)$ can be made arbitrarily close to $\phi(A, W)$.

As an immediate consequence of (10) we can see that Theorem 2.1 remains valid for the case of crisp weighting vectors, i.e. when $\gamma_i = 0$, $i = 1, \ldots, n$. In this case

$$\text{Pos}[ar{w}_i \leq A_i] = \begin{cases} 
1 & \text{if } w_i \leq a_i \\
A(w_i) & \text{if } 0 < w_i - a_i < \alpha_i \\
0 & \text{otherwise}
\end{cases}$$
where \( \bar{w}_i \) denotes the characteristic function of \( w_i \in [0, 1] \); and the weighted aggregate, denoted by \( \phi(A, w) \), is computed as
\[
\phi(A, w) = \text{Agg}\{\text{Pos}[\bar{w}_1 \leq A_1], \ldots, \text{Pos}[\bar{w}_n \leq A_n]\}
\]
If \( \text{Agg} \) is the minimum operator then we get
\[
\phi(A, w) = \min\{\text{Pos}[\bar{w}_1 \leq A_1], \ldots, \text{Pos}[\bar{w}_n \leq A_n]\}
\]
If both the ratings and the importances are given by crisp numbers (i.e. when \( \gamma_i = \alpha_i = 0, i = 1, \ldots, n \)) then \( \text{Pos}[\bar{w}_i \leq \bar{a}_i] \) implements the standard strict implication operator, i.e.,
\[
\text{Pos}[\bar{w}_i \leq \bar{a}_i] = w_i \rightarrow a_i = \begin{cases} 1 & \text{if } w_i \leq a_i \\ 0 & \text{otherwise} \end{cases}
\]
It is clear that whatever is the aggregation operator in
\[
\phi(a, w) = \text{Agg}\{\text{Pos}[\bar{w}_1 \leq \bar{a}_1], \ldots, \text{Pos}[\bar{w}_n \leq \bar{a}_n]\},
\]
the weighted aggregate, \( \phi(a, w) \), can be very sensitive to small changes in the weighting vector \( w \).

However, we can still sustain the benchmarking character of the weighted aggregation if we use an \( R \)-implication operator to transform the ratings under importance \([1, 2]\). For example, for the operator
\[
\phi(a, w) = \min\{w_1 \rightarrow a_1, \ldots, w_n \rightarrow a_n\}. \tag{13}
\]
where \( \rightarrow \) is an \( R \)-implication operator, the equation
\[
\phi(a, w) = 1,
\]
holds iff \( w_i \leq a_i \) for all \( i \), i.e. when the value of each performance rating is at least as big as the value of its associated weight. However, the crucial question here is: Does the relationship still remain valid for any \( R \)-implication?

The answer is negative. \( \phi \) will be continuous in \( w \) if and only if the implication operator is continuous. For example, if we choose the Gödel implication in (13) then \( \phi \) will not be continuous in \( w \), because the Gödel implication is not continuous.

To illustrate the sensitivity of \( \phi \) defined by the Gödel implication (4) consider (13) with \( n = 1, a_1 = w_1 = 0.6 \) and \( w_1^\delta = w_1 + \delta \). In this case
\[
\phi(a_1, w_1) = \phi(w_1, w_1) = \phi(0.6, 0.6) = 1,
\]
but
\[
\phi(a_1, w_1^\delta) = \phi(w_1, w_1 + \delta) = \phi(0.6, 0.6 + \delta) = (0.6 + \delta) \rightarrow 0.6 = 0.6,
\]
that is,
\[
\lim_{\delta \rightarrow 0} \phi(a_1, w_1^\delta) = 0.6 \neq \phi(a_1, w_1) = 1.
\]
But if we choose the (continuous) Łukasiewicz implication in (13) then \( \phi \) will be continuous in \( w \), and therefore, small changes in the importance can cause only small changes in the weighted aggregate. Thus, the following formula
\[
\phi(a, w) = \min\{(1 - w_1 + a_1) \land 1, \ldots, (1 - w_n + a_n) \land 1\}. \tag{14}
\]
not only keeps up the benchmarking character of φ, but also implements a stable approach to importance weighted aggregation in the nonfuzzy case.

If we do allow a positive compensation between ratings then we can use an OWA-operator for aggregation in (14). That is,

\[
\phi(a, w) = \text{OWA} \langle (1 - w_1 + a_1) \wedge 1, \ldots, (1 - w_n + a_n) \wedge 1 \rangle. (15)
\]

Taking into consideration that OWA-operators are usually continuous, equation (15) also implements a stable approach to importance weighted aggregation in the nonfuzzy case.

3 Illustration

In this section we illustrate our approach by several examples.

- **Crisp importance and crisp ratings**
  Consider the aggregation problem with

\[
a = \begin{pmatrix} 0.7 \\ 0.5 \\ 0.8 \\ 0.9 \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.9 \\ 0.6 \end{pmatrix}.
\]

Using formula (14) for the weighted aggregate we find

\[
\phi(a, w) = \min \{0.8 \to 0.7, 0.7 \to 0.5, 0.9 \to 0.8, 0.6 \to 0.9\} = \min \{0.9, 0.8, 0.9, 1\} = 0.8
\]

- **Crisp importance and fuzzy ratings**
  Consider the aggregation problem with

\[
a = \begin{pmatrix} (0.7, 0.2) \\ (0.5, 0.3) \\ (0.8, 0.2) \\ (0.9, 0.1) \end{pmatrix} \quad \text{and} \quad w = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.9 \\ 0.6 \end{pmatrix}.
\]

Using formula (12) for the weighted aggregate we find

\[
\phi(A, w) = \min \{1/2, 1/3, 1/2, 1\} = 1/3.
\]

The essential reason for the low performance of this object is that it performed low on the second criterion which has a high importance. If we allow positive compensations and use an OWA operator with weights, for example, \((1/6, 1/3, 1/6, 1/3)\) then we find

\[
\phi(A, w) = \text{OWA} \langle 1/2, 1/3, 1/2, 1 \rangle = 1/6 + 1/2 \times (1/3 + 1/6) + 1/3 \times 1/3 = 19/36 \approx 0.5278
\]
• *Fuzzy importance and fuzzy ratings*

Consider the aggregation problem with

\[
A = \begin{pmatrix}
(0.7, 0.2) \\
(0.5, 0.3) \\
(0.8, 0.2) \\
(0.9, 0.1)
\end{pmatrix}
\quad \text{and} \quad
W = \begin{pmatrix}
(0.8, 0.2) \\
(0.7, 0.3) \\
(0.9, 0.1) \\
(0.6, 0.2)
\end{pmatrix}.
\]

Using formula (7) for the weighted aggregate we find

\[
\phi(A, W) = \min\{3/4, 2/3, 2/3, 1\} = 2/3.
\]

The reason for the relatively high performance of this object is that, even though it performed low on the second criterion which has a high importance, the second importance has a relatively large tolerance level, 0.3.

4 Summary

We have introduced a possibilistic approach to the process of importance weighted transformation when both the importances and the aggregates are given by triangular fuzzy numbers. In our approach the importances have been considered as benchmark levels for the performances, i.e. an alternative performs well on all criteria if the degree of satisfaction to each of the criteria is at least as big as the associated benchmark. We have suggested the use of measure of necessity to be able to distinguish alternatives with overall rating one (whose performance ratings exceed the value of their importance with respect to all criteria with possibility one). We have shown that using the possibilistic approach (i) small changes in the membership function of the importances can cause only small variations in the weighted aggregate; (ii) the weighted aggregate of fuzzy ratings remains stable under small changes in the nonfuzzy importances; (iii) the weighted aggregate of crisp ratings still remains stable under small changes in the crisp importances whenever we use a continuous implication operator for the importance weighted transformation. These results have further implications in several classes of multiple criteria decision making problems, in which the aggregation procedures are rough enough to make the finely tuned formal selection of an optimal alternative meaningless. This will be the topic of a forthcoming paper.

References


5 Follow ups

The results of this paper have been improved and generalized later in the following works:

in journals

A17-c39 Hong-Bin Yan, Van-Nam Huynh, Yoshiteru Nakamori, Tetsuya Murai, On prioritized weighted aggregation in multi-criteria decision making, EXPERT SYSTEMS WITH APPLICATIONS, 38(2011), pp. 812-823. 2011
http://dx.doi.org/10.1016/j.eswa.2010.07.039

Remark. It is of interest noting that in a different context, Carlsson and Fullér (2000) concentrated on the issue of linguistic importance weighted aggregations, where the importance is interpreted as benchmarks. In their study, when both importance weights and ratings of criteria are given as crisp numbers, Łukasiewicz implication is also used to compute the benchmark achievement. In case of fuzzy numbers, a possibilistic approach is presented to compute the benchmark achievement. In addition, Carlsson and Fullér’s method only focus on symmetric triangular fuzzy numbers. The benchmark used in our aggregation operator has similar but different meaning with Carlsson and Fullér’s method (Carlsson & Fullér, 2000). Firstly, as there are various types of fuzzy numbers, specifying only symmetric triangular fuzzy numbers will not be appropriate in practical applications.
Secondly, instead of the possibility interpretation, the benchmark has a probability interpretation lying in the philosophical root of Simon’s bounded rationality (Simon, 1955) as well as represents the S-shaped value function (Kahneman & Tversky, 1979). Finally, benchmark in Carlsson and Fullér’s work (Carlsson & Fullér, 2000) and ours has different meanings. Carlsson and Fullér viewed importance weight as benchmark of criteria satisfaction, whereas our method considered DM’s requirements. (page 820)

A17-c38 Xiaohan Yu, Zeshui Xu, Xiumei Zhang, Uniformization of multigranular linguistic labels and their application to group decision making. JOURNAL OF SYSTEMS SCIENCE AND SYSTEMS ENGINEERING, 19(2010), number 3, pp. 257-276. 2010
http://dx.doi.org/10.1007/s11518-010-5137-7

In some multiple attribute decision making problems of real world, decision makers are accustomed to provide their preferences over alternatives expressed in linguistic forms, such as ”good”, ”fair”, ”poor”, etc., because of the complexity and uncertainty of practical things, and fuzzy human thinking. In order to assess alternatives using linguistic information conveniently, linguistic label sets have been researched in lots of papers (Yager 1995, 1998, Carlsson & Fullé 2000, Herrera & Herrera-Viedma 1995, Torra 1996, Xu 2004a, etc.). In the following, we briefly review some basic knowledge about linguistic label sets. (page 259)

http://dx.doi.org/10.1016/j.ijpe.2009.05.018

In the former two categories of methods, the results usually do not exactly match any of the initial linguistic terms, and then an approximation process must be developed to express the result in the initial expression domain. This produces the consequent loss of information and hence the lack of precision (Carlsson and Fuller, 2000), whereas, the third category of methods overcome the above limitations. The main advantage of this representation model is to be continuous in its domain. It can express any counting of information in the universe of the discourse. Therefore, the third one is more convenient and precise to deal with linguistic terms. (pages 550-551)

http://dx.doi.org/10.1108/17465660910943757

http://dx.doi.org/10.1002/int.20332

http://dx.doi.org/10.1007/s10726-008-9131-0
In such approach, it will either result in large-scale increases in computational complexity [14] or make the results do not exactly match any of the initial linguistic terms (and then an approximation process must be developed to express the result in the initial expression domain which induces the consequent loss of information and hence the lack of precision [A17]). (page 593)
In the former two approaches, the results usually do not match any of the initial linguistic terms, then an approximation process must be developed to express the result in the initial expression domain. This produces the consequent loss of information and hence the lack of precision [A17].

The issue of weighted aggregation has been studied extensively in, e.g., [A17], [10], [12], [22]-[24], [48], [51], and [52].


These computational techniques are as follows.

- The first one is based on the extension principle [2], [6]. It makes operations on the fuzzy numbers that support the semantics of the linguistic terms.
- The second one is the symbolic method [5]. It makes computations on the indexes of the linguistic terms.

In both approaches, the results usually do not exactly match any of the initial linguistic terms, then an approximation process must be developed to express the result in the initial expression domain. This produces the consequent loss of information and hence the lack of precision [A17]. (page 746)

In proceedings and in edited volumes

A17-c10 Jiafeng Ji; Zheng Pei, Obtaining complex linguistic rules from decision information system based genetic algorithms, IEEE International Conference on Granular Computing, 17-19 August 2009, Lushan mountain/Nanchang, China, art. no. 5255114, pp. 268-273. 2009
http://dx.doi.org/10.1109/GRC.2009.5255114

A17-c9 Van-Nam Huynh; Yoshiteru Nakamori, Group decision making with linguistic information using a probability-based approach and OWA operators, IEEE International Conference on Systems, Man and Cybernetics, 7-10 Oct. 2007, [doi 10.1109/ICSMC.2007.4413915], pp. 570-575. 2007

http://dx.doi.org/10.1007/978-3-540-49708-0_7

http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=1635846

The management of linguistic information implies the use of operators of comparison and aggregation. Many researchers have studied operators of comparison and aggregation [7] - [A17]. In [7], the linguistic weighted averaging (LWA) operator is presented as a tool to aggregate linguistic weighted information: namely, linguistic information which has associated different linguistic importance degrees. Nowadays, the fuzzy linguistic approach has been successfully applied to many different problems such as decision, information retrieval, medicine, and education etc [A17] - [15]. (page 486)
In linguistic decision analysis, irrespective of the membership function based semantics or ordered structure based semantics of the linguistic term set, one has to face the problem of weighted aggregation of linguistic information. The issue of weighted aggregation has been studied extensively in, e.g., [A17], [4], [14]. (page 482)