PDC Home Exercise 17/1: Derivation of transfer functions from block diagram

The block diagram below describes a control system for automatic steering of a ship (“Fartyg” in the block diagram). The direction of the ship is denoted by the angle \( \theta \), its measured value is \( \theta_g \) (“Givare” is the measurement device), and its setpoint is \( \theta_r \). The total torque by which the ship angle is affected is \( m \). It is composed of a torque disturbance \( m_d \) and the torque \( m_r \) delivered by the rudder, which depends on the rudder angle \( r \) through the block \( K_r \) (“O[m]vandl. Factor”). The controller \( G_c \) (“Regulator”) calculates a control signal \( u \), which affects the rudder angle through a rudder servo (“Roder servo”) \( G_r \). The input to the controller is the control error \( e = \theta_r - \theta_g \).

When the variables in the block diagram are expressed as deviation variables that express deviations from nominal operating point values, the following relationships apply:

\[
10 \ddot{\theta} + \dot{\theta} = 0.1 m; \quad m_r = 100 r; \quad 5 \dot{r} + 2 r = 0.1 u; \quad u = Ke; \quad \theta_g = \theta.
\]

Here, \( K \) is the gain of a P controller. A dot above a symbol denotes the derivative of the symbol with respect to time, two dots denote the second-order time derivative.

Derive the closed-loop transfer functions from the inputs \( \theta_r \) and \( m_d \) to the output \( \theta \), i.e., \( \Theta(s)/\Theta_r(s) \) and \( \Theta(s)/M_d(s) \), where \( \Theta(s) \), \( \Theta_r(s) \) and \( M_d(s) \) are the Laplace transforms of \( \theta(t) \), \( \theta_r(t) \) and \( m_d(t) \), respectively.