PDC 1504. A dynamical system is described by the differential equation
\[ \ddot{y}(t) + 6\dot{y}(t) + 2y(t) + 3y(t) = 2u(t) - 5\dot{u}(t) - 6u(t), \]
where \( u(t) \) denotes its input signal and \( y(t) \) its output signal as functions of time \( t \). A dot above a symbol denotes the time derivative of the quantity in question, \( n \) dots denotes the \( n \)-th order time derivative.
(a) Determine the transfer function of the system.
(b) Determine whether the system is stable or unstable.
(c) Assume that the initial conditions are zero and \( u(t) = 2, \ t \geq 0 \). What value will the output signal approach as \( t \to \infty \)?

PDC 1505. The dynamics of a system is to be determined by a step test. At the time instant 1 min, when the system is assumed to be in steady state, a step change of magnitude 2 (dimensionless unit) was made in the input signal to the system. The value of the output signal at selected time instants is given in the table below. The step response as well as the input change are also plotted in the figure on next page.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0.00</th>
<th>1.00</th>
<th>2.00</th>
<th>3.00</th>
<th>4.00</th>
<th>5.00</th>
<th>6.00</th>
<th>7.00</th>
<th>8.00</th>
<th>9.00</th>
<th>10.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.10</td>
<td>0.57</td>
<td>1.07</td>
<td>1.41</td>
<td>1.82</td>
<td>2.09</td>
<td>2.31</td>
<td>2.49</td>
</tr>
<tr>
<td>Time (min)</td>
<td>11.00</td>
<td>12.00</td>
<td>13.00</td>
<td>14.00</td>
<td>15.00</td>
<td>16.00</td>
<td>17.00</td>
<td>18.00</td>
<td>19.00</td>
<td>20.00</td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>2.57</td>
<td>2.72</td>
<td>2.84</td>
<td>2.85</td>
<td>2.89</td>
<td>2.90</td>
<td>2.94</td>
<td>2.97</td>
<td>2.98</td>
<td>2.97</td>
<td></td>
</tr>
</tbody>
</table>

Determine the following models of the system based on the step response:
(a) First-order system with possible time delay.
(b) Second-order system with possible time delay.
Check how well the models fit the data by comparing their step responses with the measured step response at the time instants 6, 10, 14 and 20 min. Submit the figure if you have used it.

PDC 1506. Consider the system described by the block diagram in the figure, where \( G_c \) denotes a controller and
\[ G_p(s) = \frac{1.5e^{-1.2s}}{8s+1}, \quad G_m(s) = e^{-0.3s}. \]
(a) Assume that \( G_c \) is a PI controller with the gain \( K_c = 3.2 \) and integral time \( T_i = 5 \) (same units as in \( G_p \) and \( G_m \)). Determine the gain margin and the phase margin of the system. Are the margins satisfactory? A Bode diagram for the loop transfer function of the system, with this controller, is plotted on next page. If you use it to find the solution, it must be submitted.
(b) The controller in case (a) is not necessarily well tuned. Tune the PI controller by some method you find suitable. Determine the gain margin and the phase margin of the system with this controller Are the margins satisfactory? Decide if you can use the submitted Bode diagram for your calculations; if not, the solution must be calculated numerically.
Step response for PDC 1505.

Bode diagram for PDC 1506.