Control of an overhead crane (direct synthesis, lead/lag design, etc.)

An overhead crane can be modelled as a pendulum hanging from a moving point. The figure shows a pendulum hanging from a point that can move in the horizontal direction. Here, $u$ is the horizontal position of the suspension point, which from now on is called the “cart”, $y$ is the horizontal position of the end point of the pendulum with mass $m$ (representing the mass of the cargo), and $\ell$ is the length of the pendulum (representing the length of the crane wire). It is desired to move the cargo from one position to another with a small amount of swinging. This means that $y$ should follow $u$ with small deviations (i.e., small angle $\theta$).

The movement of the pendulum can be described by the model

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2 + 0.1s + 1},$$

where the term $0.1s$ is due to friction. The time unit inherent in the model is seconds. The position of the cart is controlled by an electric motor whose input is a voltage signal. The transfer function of the motor is

$$G_v(s) = \frac{U(s)}{V(s)} = \frac{2}{s(s + 9)} \left[ \frac{m}{\text{volt}} \right].$$

The transfer function contains an integrator, which means that it is unstable. Therefore, a controller is needed to stabilize the cart position.

Because the cart dynamics includes an integrator, the cart position can be controlled without steady-state error by a controller that does not have integral action. Here it is appropriate to use a PD controller that uses the derivative of the cart position $u$, but not its setpoint $u_r$. The control law of the PD controller, expressed in the Laplace domain, is then

$$V(s) = K_{PD}((U_r(s) - U(s)) - T_{PD}sU(s)) = K_{PD}(U_r(s) - (1 + T_{PD}s)U(s)),$$

where $K_{PD}$ is the static gain and $T_{PD}$ is the derivative time of the controller.

1. Tune the PD controller by the method of “direct synthesis” (section 7.7 in lecture notes) to achieve a closed-loop system, comprised of the controller and the cart, with the second-order transfer function

$$G_c(s) = \frac{U(s)}{U_r(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

where the relative damping $\zeta = 0.9$ and the natural frequency $\omega_n = 10$ rad/s.

Even though the cart control is smooth, the cart movement will cause the cargo to swing because it is heavily underdamped. Therefore, it is necessary to stabilize the cargo movement by a controller connected in cascade with the cart controller. This means that the new controller (the primary controller) adjusts the setpoint of the cart controller (the secondary controller). In this case, a PID controller on series form with a derivative filter is used as primary controller as illustrated in the block diagram below, where $G_{PPIPDb}$ is the transfer function of the primary controller.
2. Design the PIPDef controller to achieve a gain-crossover frequency \( \omega_g = 2 \text{ rad/s} \) and phase margin \( \varphi_m = 30^\circ \) (section 8.4.3 in lecture notes). For the design, you can use the transfer function \( \hat{G} = G_p G_c \) from the cart setpoint \( U_r \) to the cargo position \( Y \).

3. Simulate using SIMULINK how the system reacts to a setpoint change of the cargo position. Use a step change as setpoint change \( y_r \) (e.g., 1 m) and plot how the cart position \( u \) and the cargo position \( y \) change with time. Note that you should simulate the system composed of \( G_p, G_v, G_{PD} \) and \( G_{PIPDf} \) connected as in the block diagram (i.e., you should note simulate with the design model \( G_c \)).