5. Dynamics of Simple Systems

5.3 Second-order systems

5.3.2 Identification of overdamped system

An overdamped second-order system without zeros, but possibly including a time-delay, can be identified from its step response by a simple “graphical” technique. Such a system has the transfer function

\[ G(s) = \frac{Ke^{-Ls}}{(T_1s+1)(T_2s+1)} , \quad T_1 \geq T_2 \geq 0 \]  

(5.3.1)

where

- \( K \) is the static gain
- \( T_1 \) and \( T_2 \) are time constants
- \( L \) is a time delay

The assumption \( T_1 \geq T_2 \geq 0 \) is only for convenience. As special cases it includes

- \( T_1 = T_2 > 0 \), i.e. a critically damped system
- \( T_1 > 0, T_2 = 0 \), i.e. a first-order system
Harriott’s method (slightly modified)

Plot the step response

- the size of the input step is $u_{\text{step}}$ 
- the final change of the output is $y_\infty$
- calculate $K = y_\infty / u_{\text{step}}$
- the time it takes to reach $0.72y_\infty$ is $t_{72}$
- estimate the time delay $L$ “visually” (do not use tangent method)
- calculate $t_z = 0.4t_{72} + 0.6L$
- the value of the output at time $t_z$ is $y_z$; find it from the step response
- calculate $y_z / y_\infty$ and read $z$ from this diagram
  - if $y_z / y_\infty < 0.27$, the time delay $L$ has to be increased
  - if $y_z / y_\infty > 0.4$, $L$ has to be decreased
  - if a new value for $L$ was selected, calculate a new $t_z$ etc.
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- read $\tau_{72}$ corresponding to $z$ from this diagram
- calculate $T_{\Sigma} = (t_{72} - L)/\tau_{72}$
- calculate
  - $T_1 = zT_{\Sigma}$
  - $T_2 = T_{\Sigma} - T_1$
- now all parameters of the transfer function are known!