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2. Basic Control Concepts

2.1 Signals and systems

A system can be defined as a combination of components that act together to perform a certain objective.

A system interacts with its environment through signals.

There are two main types of signals:

- **input signals (inputs) \( u \)**, which affect the system behaviour in some way
- **output signals (outputs) \( y \)**, which give information about the system behaviour

There are two types of input signals:

- **control signals** are inputs whose values we can adjust
- **disturbances** are inputs whose values we cannot affect (in a rational way)

Generally, signals are functions of time \( t \), which can be indicated by \( u(t) \) and \( y(t) \).
2. Basic Control Concepts

2.1 Signals and systems

A signal is (usually) a *physical quantity or variable*. Depending on the context, the term “signal” may refer to the

- type of variable (e.g. a variable denoting a temperature)
- value of a variable (e.g. a temperature expressed as a numerical value)

In practice, this does *not* cause confusion.

The value of a signal may be known if it is a *measured variable*. In particular,

- some *outputs* are (nearly always) measured
- some *disturbances* might be measured
- *control signals* are either measured or known because they are given by the controller

A system is a

- *static system* if the outputs are completely determined by the inputs at the same time instant; such behaviour can be described by *algebraic equations*
- *dynamic(al) system* if the outputs depend also on inputs at previous time instants; such behaviour can be described by *differential equations*
Example 2.1. Block diagram of a control valve.

Figure 2.2 illustrates a control valve.

- The flow $q$ through the control valve depends on the valve position $x$, primary pressure $p_1$ and secondary pressure $p_2$.

- The valve characteristics (provided by the valve manufacturer) give a relationship between the steady-state values of the variables. In reality, the flow $q$ depends on the other variables in a dynamic way.

- The flow $q$ is the output signal of the system, whereas $x$, $p_1$ and $p_2$ are input signals.

- Of the input signals, $x$ can be used as a control signal, while $p_1$ and $p_2$ are disturbances.

Fig. 2.2. Schematic of a control valve.  
Fig. 2.3. A block diagram.
2. Basic Control Concepts

2.2 Block diagrams

A **block diagram** is a

- pictorial representation of *cause-and-effect relationships* between signals.

The signals are represented by

- *arrows*, which show the *direction of information flow*.

In particular, a block with signal arrows denotes that

- the *outputs of a dynamical system depend on the inputs*.

The simplest form a block diagram is a single block, illustrated by Fig. 2.1.

The interior of a block usually contains

- a *description* or the *name* of the corresponding system, or
- a *symbol for the mathematical operation* on the input to yield the output.

![Diagram](image)

**Fig. 2.4. Examples of block labeling.**
2. Basic Control Concepts

2.2 Block diagrams

The blocks in a block diagram consisting of several blocks are connected via their signals. The following algebraic operations on signals of the same type are often needed:

- **addition**

  ![Addition Diagram]

- **subtraction**

  ![Subtraction Diagram]

- **branching**

  ![Branching Diagram]
Figure 2.5 shows symbols for flow control in a process diagram:

- “FC” is a flow controller
- “FT” is a flow transmitter

The notations “FIC” and “FIT” are also used, where “I” indicates that the instrument is equipped with an “indicator” (analog or digital display of data).

Other common examples of notation are

- “LC” for level controller
- “TC” for temperature controller
- “PC” for pressure controller
- “QC” for concentration controller

Fig. 2.5. Process diagram for flow control.
2. Basic Control Concepts

2.3 From flow sheet to block diagram

Note the following for input and output signals used in control engineering.

- The input and output signals in a control system block diagram are *not equivalent to the physical inlet and outlet currents* in a process flow diagram.
- The *input signals* in a control system block diagram indicate which variables *affect* the system behaviour while the *output signals give information* about the system behaviour.
- The input and output signals in control systems are *not necessarily streams in a literal sense*, and even if they are, *the signal direction does not have to be the same as the direction of the corresponding physical stream*. For instance, a physical outlet stream may well be a control input signal as shown in Ex. 2.2 on next slide.
- The output signals in a block diagram provide *some information about the purpose of the process*, which cannot be directly understood from a process flow diagram. Usually the choice of the control signals and the presence of disturbances are not unambiguously apparent from the process flow diagram. In other words, the block diagram provides better information for process control than a process flow diagram.
Example 2.2. Block diagram of a tank with continuous throughflow.

**Process A.** A liquid tank, where the fluid level $h$ can be controlled by the inflow $F_1$, and the outflow $F_2$ depends on $h$ (discharge by gravity).

Block diagram:

**Process B.** A liquid tank, where the fluid level $h$ can be controlled by the outflow $F_2$, and the inflow $F_1$ is a disturbance variable.

Block diagram:

The block diagram also illustrates what is meant by a positive and negative gain.
Exercise 2.1.

Design a block diagram for the following process, where a liquid flowing through a tube is heated by introduction of steam into the tube. The temperature of the heated liquid is controlled by the flow rate of steam.

\[ v = 1 \text{ m/s} \]

\[ 60 \text{ m} \]

\[ \vartheta_i \rightarrow \vartheta_1 \rightarrow \vartheta_r \rightarrow \vartheta_2 \]

liquid

steam

TC

v

2. Basic Control Concepts

2.4 Control strategies

2.4.1 Open-loop control

In some simple applications, *open-loop control without measurements* can be used. In this control strategy:

- the controller is tuned using some *a priori information* (a “model”) about the process
- after the tuning has been made, the control action is a function of the *setpoint only* (setpoint = desired value of the controlled variable)

This control strategy has *some advantages*, but also *clear disadvantages*. *Which?*

*Examples* of open-loop control applications:

- bread toaster
- idle-speed control of (an old) car engine

![Fig. 2.6. Open-loop control.](image-url)
2. Basic control concepts

2.4 Control strategies

2.4.2 Feedforward control

Control is clearly needed to eliminate the effect of disturbances on the system output. *Feedforward control* is a type of open-loop control strategy, which can be used for disturbance elimination, *if*

- disturbances can be measured
- it is known how the disturbances affect the output
- It is known how the control signal affects the output

Feedforward is an open-loop control strategy because *the output*, which we want to control, *is not measured*.

Obviously, this control strategy has *advantages*, but it also has some disadvantage. *Which?*

When feedforward control is used, it is usually used *in combination with feedback control*.

![Fig. 2.7. Feedforward control.](image)
2. Basic control concepts

2.4 Control strategies

2.4.3 Feedback control

Generally, successful control requires that an output variable is measured. In feedback control, this measurement is fed to the controller. Thus

- the controller receives information about the behaviour of the process
- usually, the measured variable is the variable we want to control (in principle, it can also be some other variable)

![Feedback control diagram](image)

**Fig. 2.8. Feedback control.**
Example 2.3. Two different control strategies for house heating.

The figures below illustrates the heating of a house by (a) feedforward, (b) feedback control. Some advantages and disadvantages can be noted:

- **Feedforward:** Rapid control because the controller acts before the effect of the disturbance (outdoor temperature) is seen in the output signal (indoor temperature), but requires good knowledge of the process model; does not consider other disturbances (e.g. the wind speed) than the measured outside temperature.

- **Feedback:** Slower control because the controller does not act before the effect of the disturbance (outdoor temperature) is seen in the output signal (indoor temperature); less sensitive to modelling errors and disturbances.

What would open-loop control of the indoor temperature look like?

(a) feedforward   (b) feedback
Exercise 2.2.

Consider the two flow control diagrams below. Indicate the control strategies (feedback or feedforward) in each case and justify the answer. It can be assumed that the distance between the flow transmitter FT and the control valve is small.

System A

System B
Exercise 2.3.

The liquid tank to the right has an inflow $F_1$ and an outflow $F_2$. The inflow is controlled so that $F_1 = 10$ l/min.

The volume of the liquid is desired to remain constant at $V = 1000$ liters. The volume of the liquid (or the liquid level) is thus the output signal of the system, whereas $F_1$ and $F_2$ are input signals.

The following control strategies are possible:

a) Open-loop control – the outflow is measured and controlled so that $F_2 = 10$ l/min.

b) Feedforward – the inflow is measured and the outflow is controlled so that $F_2 = F_1$.

c) Feedback – the liquid level $h$ is measured and controlled by the outflow $F_2$.

Discuss the differences between these strategies and propose a suitable strategy.
2. Basic control concepts

2.4 Control strategies

a) 

b) 

c)
2. Basic control concepts

2.5 Feedback control

2.5.1 The basic feedback structure

Figure 2.9 shows a block diagram of a simple closed-loop control system.

- The objective of the control system is to control the measured output signal \( y \) (a single variable) of the controlled system to a desired value, also called a setpoint or reference value.

- Normally, the controller operates directly on the difference between the setpoint \( r \) and the measured value \( y_m \) of the output signal \( y \), i.e. on the control deviation or control error.

- The output signal (at a certain instant) is sometimes called actual value.

![Block diagram of a simple closed-loop control system.](image)

**Fig. 2.9. Standard feedback control structure.**
Two types of control can be distinguished depending on whether the setpoint is mostly constant or changes frequently:

- **Regulatory control.** The setpoint is usually constant and the main objective of the control system is to maintain the output signal at the setpoint, despite the influence of disturbances. This is sometimes referred to as a *regulatory problem*.

- **Tracking control.** The setpoint varies and the main objective of the control system is to make the output signal follow the setpoint with as little error as possible. This is sometimes referred to as a *servo problem*.

These two types of control tasks may well be handled simultaneously; the differences arise in the choice of parameter values for the controller (Chapter 7).
2. Basic control concepts

2.5 Feedback control

2.5.2 An example of what can be achieved by feedback control

We shall illustrate some fundamental properties of feedback control by considering control of the inside temperature of a house.

The temperature \( \vartheta_i \) inside the house depends on the outside temperature \( \vartheta_a \) and the heating power \( P \) according to some dynamic relationship. If we assume that

- \( \vartheta_i \) depends linearly (or more accurately, affinely) on \( P \)
- the dynamics are of first order

the relationship between the variables can be written

\[
T \frac{d\vartheta_i}{dt} + \vartheta_i = K_p P + \vartheta_a
\]  

(2.1)

where \( K_p \) is the static gain and \( T \) is the time constant of the system. The system parameters have the following interpretations:

- \( K_p \) denotes how strong the effect of a system input (in this case \( P \)) is on the output; a larger value means a stronger effect.
- \( T \) denotes how fast the dynamics are; a larger value means a slower system.
2.5 Feedback control

2.5.2 ... what can be achieved by feedback control

In this case, $K_p > 0$. Equation (2.1) shows that in the steady-state ($d\vartheta_i/dt = 0$)

- $\bar{\vartheta}_i = \bar{\vartheta}_a$ if $\bar{P} = 0$
- an increase of $\bar{P}$ increases $\bar{\vartheta}_i$
- an increase of $\bar{\vartheta}_a$ increases $\bar{\vartheta}_i$

Thus, the simple model (2.1) has the same basic properties as the true system. We want

- the inside temperature $\bar{\vartheta}_i$ to be equal to a desired temperature $\vartheta_r$
- in spite of variations in the outside temperature $\bar{\vartheta}_a$
- even if the system gain $K_p$ and time constant $T$ are not accurately known.

A simple control law is to adjust the heating power in proportion to the difference between the desired and the actual inside temperature, i.e.,

$$\bar{P} = K_c(\vartheta_r - \bar{\vartheta}_i) + P_0$$

(2.2)

where $K_c$ is the controller gain and $P_0$ is a constant initial power, which can be set manually. This relationship describes a proportional controller, more commonly known as a P-controller.

If $K_c > 0$, the controller has the ability to increase the heating power when the inside temperature is below the desired temperature.
2.5 Feedback control  

By combining equation (2.1) and (2.2), we can get more explicit information about the controlled system behaviour. Elimination of the control signal \( P \) gives

\[
\bar{\vartheta}_i = \frac{K_p K_c}{1 + K_p K_c} \vartheta_r + \frac{1}{1 + K_p K_c} \bar{\vartheta}_a + \frac{K_p}{1 + K_p K_c} P_0.
\]  

(2.3)

From this equation we can deduce the following:

- If the temperature control is turned off so that \( K_c = 0 \), we get \( \bar{\vartheta}_i = \bar{\vartheta}_a + K_p P_0 \), i.e. the inside temperature is not a function of the desired temperature \( \vartheta_r \).

- If, in addition to \( K_c = 0 \), the initial heating power is turned off so that \( P_0 = 0 \), the inside temperature will be equal to the outside temperature.

- If the controller is set in automatic mode (\( K_c = 0 \)) and \( K_c = 1/K_p \) is chosen, we get \( \bar{\vartheta}_i = 0.5 \vartheta_r + 0.5 \bar{\vartheta}_a + 0.5 K_p P_0 \), i.e. the inside temperature will be closer to the desired temperature than the outside temperature (if \( \vartheta_r > \bar{\vartheta}_a \)).

- Depending on the value of \( P_0 \), we might even obtain \( \bar{\vartheta}_i = \vartheta_r \) for some value of \( \bar{\vartheta}_a \).
It is easy to see that

- the higher $K_c$ is, the more $\bar{\vartheta}_i$ approaches the reference value $\vartheta_r$, independently of $\bar{\vartheta}_a$ and $P_0$; i.e. $\bar{\vartheta}_i \to \vartheta_r$ if $K_c \to \infty$.

Thus, the following are fundamental properties of feedback control:

- It can *almost completely eliminate the effect of disturbances* (the outside temperature $\bar{\vartheta}_a$ in this example) on the controlled system.

- Normally, we *do not need to know the characteristics of the system in detail* ($K_p$ in this example) in order to tune the controller.

- We can *make the output signal stay at or follow a desired value* ($\bar{\vartheta}_i \approx \vartheta_r$ in this example).
2. Basic control concepts

2.5 Feedback control

2.5.3 A counter-example: limiting factors

In the example above, we neglected the system dynamics in order to illustrate in a simple way the advantages that, at least in principle, can be achieved by feedback control.

- It is clear, for example, that in practice we cannot have a controller gain that approaches infinity.

- Even if this were possible, equation (2.2) would then require an input power which approaches infinity if the inside temperature deviates from the reference temperature. Of course, such a power is not available.

- In addition, the properties of the system to be controlled generally limit the achievable control performance. This is illustrated by the following example.
Consider the process in Exercise 2.1, where the fluid flowing in a well-insulated tube is heated and the temperature is controlled by direct addition of steam.

The temperature $\theta_2$ of the liquid is measured 60 m after the mixing point. Because the flow velocity $v = 1$ m/s, this means that the temperature $\theta_1$ at the mixing point reaches the measuring point 1 min later.

If the liquid temperature before the mixing point is denoted $\theta_i$ and the mass flow rate of the added steam is denoted $\dot{m}$, the following expression applies when the heat loss from the tube is neglected:

$$\theta_2(t + 1) = \theta_1(t) = \theta_i(t) + K_p \dot{m}(t).$$  \hspace{1cm} (2.4)

Here $t$ is time expressed in minutes and $K_p$ is a positive process gain.

A P-controller is used for control of $\theta_2$ with $\dot{m}$ (the control valve is neglected). Then

$$\dot{m}(t) = K_c (\theta_r - \theta_2(t)) + \dot{m}_0$$  \hspace{1cm} (2.5)

where $K_c$ is the controller gain and $\dot{m}_0$ is a constant value of the mass flow rate of steam, which is chosen to yield $\bar{\theta}_2 \approx \theta_r$ at the normal steady state.
Combining equations (2.4) and (2.5) gives
\[ \vartheta_2(t + 1) = \vartheta_i(t) + K_p K_c (\vartheta_r - \vartheta_2(t)) + K_p \dot{m}_0. \] (2.6)

Consider a steady state \((\bar{\vartheta}_i, \bar{\vartheta}_2)\). According to equation (2.6), the following expression then applies:
\[ \bar{\vartheta}_2 = \bar{\vartheta}_i + K_p K_c (\vartheta_r - \bar{\vartheta}_2) + K_p \dot{m}_0. \] (2.7)

Subtracting equation (2.7) from (2.6) yields
\[ \Delta \vartheta_2(t + 1) = \Delta \vartheta_i(t) - K_p K_c \Delta \vartheta_2(t) \] (2.8)
where \( \Delta \vartheta_i(t) \equiv \vartheta_i(t) - \bar{\vartheta}_i \) and \( \Delta \vartheta_2(t) \equiv \vartheta_2(t) - \bar{\vartheta}_2 \).

Assume that steady-state conditions apply up to time \( t = 0 \), and that a sudden change of size \( \Delta \vartheta_{i,\text{step}} \) occurs in the temperature \( \vartheta_i \) at this time. According to equation (2.8) this results in \( \Delta \vartheta_2(1) = \Delta \vartheta_{i,\text{step}}, \)
\( \Delta \vartheta_2(2) = \Delta \vartheta_{i,\text{step}} - K_p K_c \Delta \vartheta_2(1) = (1 - K_p K_c)\Delta \vartheta_{i,\text{step}}, \) etc. for \( t > 1 \).

The general expression for \( t = k \) becomes
\[ \Delta \vartheta_2(k) = \sum_{j=0}^{k-1} (-K_p K_c)^j \Delta \vartheta_{i,\text{step}}. \] (2.9)
We see immediately:

- If \( |K_pK_c| > 1 \), the absolute value of every term on the right-hand side of eq. (2.9) is greater than the previous term (with smaller \( j \)), i.e., the series diverges, which results in instability.

- If \( K_pK_c = 1 \), \( \Delta \theta_2 \) will “for ever” oscillate between the levels \(-\Delta \theta_{i,\text{step}}\) and \(+\Delta \theta_{i,\text{step}}\).

- If \( |K_pK_c| < 1 \), the sum of all terms form a converging geometric series, and we get

\[
\Delta \theta_2(k) \to \frac{\Delta \theta_{i,\text{step}}}{1+K_pK_c} \quad \text{when} \quad k \to \infty, \quad |K_pK_c| < 1
\]  

(2.10)

The expression (2.10) shows that the best control with a P-controller yields

\[
\Delta \theta_2(k) \approx 0.5\Delta \theta_{i,\text{step}} \quad \text{when} \quad k \to \infty
\]

although we would desire \( \Delta \theta_2(k) \approx 0 \).
2.5 Feedback control

2.5.3 A counter-example: limiting factors

In this example, we did not obtain the very positive effects we did obtain in the example before.

- The process is not especially complicated, but it has a pure transport delay, or more generally, a time delay, also called dead time.
- Such transport delays are very common in the process industry, but even other processes often have time delays.
- In general we can say that a time delay in a feedback control system can have very harmful effects on the performance of the closed-loop control, and it can even compromise the control-loop stability.

Time delays are troublesome characteristics of a process, but some processes can also be difficult to control due to other factors.

- For example, processes whose behaviour is described by (linear) differential equations of third order or higher have restrictions and performance limitations of similar type as the ones caused by time delays.
2.5 Feedback control

2.5.4 The PID controller

In the two examples above a P-controller was used and we established the following:

- A high controller gain is desirable for elimination of the influence of external disturbances on the controlled system, and also for reduction of the sensitivity to uncertainty in the process parameters.

- A high gain may cause instability, and the situation is aggravated by process uncertainties; one can say that the risk of instability is imminent if one relies too much on old information.

- A stationary control deviation (a lasting control error) is obtained after a load change (i.e., a disturbance); the smaller the controller gain is, the larger the error.

The first two items apply to feedback control in general.

- Since they are mutually contradictory, they suggest that compromises must be made in order to find an optimal controller tuning.

- It is likely that a more complex controller than a P-controller should be used. This is necessary e.g. for elimination of a stationary control deviation.
The so-called PID controller is a "universal controller", which in addition to a pure gain, also contains an integrating part and a derivative part. The control law of an ideal PID controller is given by

$$u(t) = K_c \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right) + u_0$$  \hspace{1cm} (2.11)

Here $u(t)$ is the output signal of the controller and $e(t)$ is the difference between the reference value and the measured value, i.e. the control error; see Figure 2.9.

The adjustable parameters of the controller are, in addition to the initial output value $u_0$ (often = 0),

- the controller gain $K_c$
- the integral time $T_i$
- the derivative time $T_d$
2.5 Feedback control

2.5.4 The PID controller

By choosing appropriate controller parameters, parts of the controller that are not needed can be disabled.

- A so-called **PI-controller** is obtained by letting $T_d = 0$.
- A **P-controller** is obtained by $T_i = \infty$ (not $T_i = 0$ !) and $T_d = 0$.
- Sometimes **PD-controllers** are used.

In practice, a P-effect is always required, and as the control law is written in equation (2.11), it cannot be disabled (by letting $K_c = 0$) without disabling the whole controller. This limitation can be eliminated by writing the control law in a so-called **non-interacting form**

$$u(t) = K_c e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} + u_0$$

(2.12)

Now, for example, a pure integral controller is obtained by letting $K_c = 0$ and $K_d = 0$.

In practice, there are many other modifications of the ideal PID controller. See Chapter 7.
The **PI-controller** is without doubt the **most common controller** in the (process) industry, where it is especially used for flow control. The PI-controller has

- good static properties, because it *eliminates stationary control deviation*;
- a tendency to cause *oscillatory behaviour*, which reduces the stability (the integral collects old data!).

The **D-effect** is often included (PD or PID) in the control of **processes with slow dynamics**, especially temperature and vapour pressure. The D-effect gives

- good dynamic properties and *good stability* (the derivative “predicts” the future!);
- *sensitivity to measurement noise*. 
2.5 Feedback control

2.5.4 The PID controller

Exercise 2.4.
Consider a PI-controller and assume that steady-state conditions apply for times \( t \geq t_s \). This means that \( u(t) \) and \( e(t) \) are constant (\( = u(t_s) \) and \( e(t_s) \), respectively) for \( t \geq t_s \).

Explain why this implies that \( e(t_s) = 0 \), i.e., that the control deviation must be zero at steady state.

Exercise 2.5.
Consider the double-integral controller (PII controller)

\[
u(t) = K_c \left( e(t) + \frac{1}{T_i} \int_0^t x(\tau) \, d\tau \right) + u_0 , \quad x(t) = \int_0^t e(\tau) \, d\tau .
\]

What steady-state properties does this controller have, i.e., what can be said about \( e(t) \) and/or \( x(t) \) at steady state?

2.5.5 Negative and positive feedback

It is important to distinguish between negative feedback and positive feedback.

- **Negative feedback** means that the control signal reduces the control error.
- **Positive feedback** means that the control signal amplifies the control error.
Exercise 2.6.

1. What kind of feedback — positive or negative — should be used in a control system?
2. How do you know what kind of feedback you have in a control system?
3. Is it always possible to choose the right type of feedback?
4. What happens if the wrong type of feedback is chosen?

Often other definitions of negative (and positive) feedback are mentioned in the control engineering literature, for example:

- Negative feedback means that the control signal increases when the output signal decreases, and vice versa.
- Negative feedback is obtained when the measured value of the output signal is subtracted from the setpoint.

5. Are these definitions in accordance with the definitions given in Section 2.5.5?
6. If not, what is required concerning the signs of the process and/or controller gains for these definitions to be equivalent to those in Section 2.5.5?