Facilitating construction of safety cases from formal models in Event-B

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Abstract

Context: Certification of safety–critical software systems requires submission of safety assurance documents, e.g., in the form of safety cases. A safety case is a justification argument used to show that a system is safe for a particular application in a particular environment. Different argumentation strategies (informal and formal) are applied to determine the evidence for a safety case. For critical software systems, application of formal methods is often highly recommended for their safety assurance.

Objective: The objective of this paper is to propose a methodology that combines two activities: formalisation of system safety requirements and the demonstration of their fulfilment. The proposed methodology is illustrated by numerous case studies.

Method: We propose a classification of system safety requirements in order to facilitate the mapping of informally defined requirements into a formal model. Moreover, we propose a set of argument patterns that aim at enabling the construction of (a part of) a safety case from a formal model in Event-B.

Results: The results reveal that the proposed classification-based mapping of safety requirements into formal models facilitates requirements traceability. Moreover, the provided detailed guidelines on construction of safety cases aim to simplify the task of the argument pattern instantiation for different classes of system safety requirements. The proposed methodology is illustrated by numerous case studies.

Conclusion: Firstly, the proposed methodology allows us to map the given system safety requirements into elements of the formal model to be constructed, which is then used for verification of these requirements. Secondly, it guides the construction of a safety case, aiming to demonstrate that the safety requirements are indeed met. Consequently, the argumentation used in such a constructed safety case allows us to support it with formal proofs and model checking results used as the safety evidence.

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1. Introduction

Safety–critical software systems are subject to certification. More and more standards in different domains require submission of safety cases as a part of the safety assurance process of such systems, e.g., ISO 26262 [1], EN 50128 [2], and the UK Defence Standard 00-56 [3]. Safety cases are justification arguments for safety. They justify why a system is safe and whether the design adequately incorporates the safety requirements defined in a system requirement specification to comply with the safety standards. To facilitate the construction of safety cases, two main graphical notations have been proposed: Claims, Arguments and Evidence (CAE) notation [4] and Goal Structuring Notation (GSN) [5]. In our work, we rely on the latter one due to its support for argument patterns, i.e., common structures capturing successful argument approaches that can be reused within a safety case [6]. To demonstrate the compliance with the safety standards, different types of evidence can be used [7]. Among them are results of hazard analysis, testing, simulation, formal verification, manual inspection, etc.

At the same time, the use of formal methods is highly recommended for certification of safety–critical software systems [8]. During the last two decades, these methods have been successfully applied to development and verification of systems from various domains. Experience in the practical use of formal methods is summarised in [9,10]. The issues related with formalisation of safety requirements and the demonstration of their fulfilment are becoming more important in industry and can be, to some extent, addressed by formal methods. In this paper we focus more on the first one – formalisation of safety requirements, which can lead to identification of contradicting or incomplete requirements. Nevertheless, small cores of highly-critical systems can be also...
formally developed and verified. The program code is then generated from detailed formal models.

In general, formal modelling and verification can give the system developers extra assurance (e.g., via theorem proving covering all the executions paths) on the system safety properties to be preserved. In particular, theorem proving gives us much stronger guarantees that a property holds in comparison to model checking or testing, provided that the formal models are a faithful representation of a system. There are several works dedicated to show how formal proofs can contribute to a safety case, e.g., [11–15]. For instance, such approaches as [11,12] apply formal methods to ensure that different types of safety properties of critical systems hold while focusing on particular blocks of software system implementation (C code). The authors of [15] propose a generic approach to automatic transformation of the formal verification output into a software safety assurance case. Similarly to [11,12], a formalised safety requirement in [15] is verified to hold at a specific location (a specific line number for code, a file, etc.).

In our work, we deal with formal system models rather than the code. A high level of abstraction allows us to cope with the complexity of computer-based systems yet ensuring the desired safety properties. Formal models are more concise, which makes it easier to trace all the places associated with a particular requirement. Formal verification (based on theorem proving) allows us to check all the possible execution paths to find, e.g., conflicting requirements. Overall, formal modelling and verification gives us valuable feedback on early design stages and thus facilitates creating cleaner architectures as well as structuring the system design.

We rely on formal modelling techniques, including external tools that can be bridged together, that are scalable to analyse the entire system. Our chosen formal framework is Event-B [16] – a state-based formal method for system level modelling and verification. Event-B aims at facilitating modelling of parallel, distributed and reactive systems. Recently, it has been actively used within large EU FP7 projects (RODIN [17], DEPLOY [18], ADVANCE [19]) to model and verify complex systems. Scalability in Event-B can be achieved via abstraction, proof and decomposition. Moreover, this formalism has strictly defined semantics and mature tool support – the Rodin platform [20] accompanied by various plug-ins, including the ones for program code generation, e.g. C, Java, etc. This allows us to model and verify a wide range of different safety-related properties stipulated by the given system safety requirements. Those requirements may include safety requirements about global and local system properties, the absence of system deadlocks, temporal and timing properties, etc. Software safety requirements can be derived from the given system safety requirements. Moreover, the systems approach that we rely on supports (advocates) derivation of controlling software from the overall system model. This allows us to identify intricate interdependencies between system software and the underlying hardware components.

In this paper, we significantly extend and exemplify with a number of case studies our previous work [21] on linking modelling in Event-B with safety case construction. More specifically, we further elaborate on the classification of safety requirements and define how each class can be treated formally to allow for verification of the given safety requirements, i.e., we define *mapping* of the classified safety requirements into the corresponding elements of Event-B. The Event-B semantics then allows us to associate them with particular theorems (proof obligations) to be proved when verifying the system. The employed formal framework assists the developers in automatic generation of the respective proof obligations. This allows us to use the obtained proofs as the evidence, showing that the given requirements fitting into our classification are fulfilled, in system safety cases. Finally, to facilitate the construction of safety cases, we define a number of argument patterns where the argumentation and goal decomposition in safety cases are based on the results obtained from the associated formal reasoning.

Therefore, the overall contribution of this paper is a developed methodology that covers two main processes: (1) integration of formalised system safety requirements into formal models of software systems, and (2) construction of structured safety cases from such formal models. Here the term “a structured safety case” stands for a safety case constructed using the associated graphical notation, e.g., GSN, to structure safety arguments.\(^1\) The proposed methodology is illustrated by six case studies, namely, a steam boiler control system [22,23], a sluice gate control system [24], an attitude and orbit control system [25], a system for controlling cars on a bridge [16], a temperature monitoring system [26], and a data processing unit of Mercury planetary orbiter of the BepiColombo mission [27,28]. The majority of the used case studies came from European and national industrial projects we have been involved in [17,18]. The extended version of this paper is available as a technical report [29].

The remainder of the paper is structured as follows. In Section 2, we briefly introduce our modelling framework – Event-B, its refinement-based approach to modelling software systems as well as the Event-B verification capabilities based on theorem proving. Additionally, we overview the notion of safety cases and their supporting graphical notation. Section 3 describes our methodology for rigorous construction of safety cases and proposes a classification of safety requirements. We elaborate on the proposed methodology in Section 4, where we define a set of argument patterns and their verification support. We also exemplify pattern instantiation on a set of small case studies. Section 5 summarises the presented methodology and discusses its validation, while Section 6 overviews the related work. Finally, in Section 7, we give concluding remarks as well as discuss our future work.

2. Preliminaries

In this section, we briefly outline the Event-B formalism that we use to derive models of safety–critical systems. In addition, we briefly describe the notion of safety cases and their supporting notation that we will rely on in this paper.

2.1. Overview of Event-B

**Event-B language.** Event-B [16,20] is a state-based formal method for system level modelling and verification. It is a variation of the B Method [30]. Automated support for modelling and verification in Event-B is provided by the Rodin platform [20].

Formally, an Event-B model is defined by a tuple \( (d, c, A, v, \Sigma, I.\ Init, E) \), where \( d \) stands for sets (data types). \( c \) are constants, \( v \) is a vector of model variables, \( \Sigma \) corresponds to a model state space defined by all possible values of the vector \( v \). \( A(d, c, v) \) is a conjunction of axioms defining properties of model data structures, while \( I(d, c, v) \) is a conjunction of invariants defining model properties to be preserved. \( \text{Init} \) is an non-empty set of model initial states, \( \text{Init} \subseteq \Sigma \). Finally, \( E \) is a set of model events where each event \( e \) is a relation of the form \( e \subseteq \Sigma \times \Sigma \).

The sets and constants of the model are stated in a separate component called CONTEXT, where their properties are postulated as axioms. The model variables, invariants and events, including initialisation event, are introduced in the component called MACHINE. The model variables are strongly typed by the constraining predicates in terms of invariants.

In general, an event \( e \) has the following form:

\[ e \models \text{any} \; I \; v \; \text{where} \; g \; \text{then} \; R \; \text{end.} \]

\(^1\) From now on, by safety cases we always mean structured safety cases.
where \(lv\) is a list of local variables, the guard \(g\) is a conjunction of predicates defined over the model variables, and the action \(R\) is a parallel composition of assignments over the variables.

The event guard defines when an event is enabled. If several events are enabled simultaneously then any of them can be chosen for execution non-deterministically. If none of the events is enabled then the system deadlocks. In general, the action of an event is a composition of assignments executed simultaneously.

Variable assignments can be either deterministic or non-deterministic. The deterministic assignment is denoted as \(x := \text{Expr}(v)\), where \(x\) is a state variable and \(\text{Expr}(v)\) is an expression over the state variables \(v\). The non-deterministic assignment can be denoted as \(x := S \lor x \lor Q(v,x)\), where \(S\) is a set of values and \(Q(v,x)\) is a predicate. As a result of the non-deterministic assignment, \(x\) gets any value from \(S\) or it obtains a value \(x'\) such that \(Q(v,x')\) is satisfied.

The Event-B language can also be extended by different kinds of attributes attached to model events, guards, variables, etc. We will use Event-B attributes to contain formulas or expressions to be used by external tools or Rodin plug-ins, e.g., Linear Temporal Logic (LTL) formulas to be checked.

**Event-B semantics.** The semantics of Event-B events is defined using before-after predicates [31]. A before-after predicate \(BA\) describes a relationship between the system states before and after execution of an event. Hence, the definition of an event presented above can be given as the relation describing the corresponding state transformation from \(v\) to \(v'\), such that:

\[
e(v,v') = g_e(v) \land I(v) \land BA_e(v,v'),
\]

where \(g_e\) is the guard of the event \(e\), \(BA_e\) is the before-after predicate of this event, and \(v, v'\) are the system states before and after event execution respectively.

Sometimes, we need to explicitly reason about possible model states before or after some particular event. For this purpose, we introduce two sets — before \(e\) and after \(e\). Specifically, before \(e\) represents a set of all possible pre-states defined by the guard of the event \(e\), while after \(e\) is a set of all possible post-states of the event \(e\), i.e., \(e, after(e) \subseteq \Sigma\) denote the domain and range of the relation \(e\) [32]:

\[
\text{before}(e) = \{v \in \Sigma \mid I(v) \land g_e(v)\},
\]

\[
\text{after}(e) = \{v' \in \Sigma \mid I(v') \land \exists v \in \Sigma. I(v) \land g_e(v) \land BA_e(v,v')\}.
\]

To verify correctness of an Event-B model, we generate a number of proof obligations (POs). More precisely, for an initial (i.e., abstract) model, we prove that its initialisation and all events preserve the invariants:

\[
A(d,c). I(d,c,v) \land g_e(d,c,v) \land BA_e(d,c,v,v') \Rightarrow I(d,c,v').
\]  

(INV)

Since the initialisation event has no initial state and guard, its proof obligation is simpler:

\[
A(d,c). \text{BA}_{\text{INIT}}(d,c,v') \Rightarrow I(d,c,v').
\]  

(INIT)

On the other hand, we verify event feasibility. Formally, for each event \(e\) of the model, its feasibility means that, whenever the event is enabled, its before-after predicate is well-defined, i.e., there exists some reachable after-state:

\[
A(d,c). I(d,c,v) \land g_e(d,c,v) \Rightarrow \exists v'. BA_e(d,c,v,v').
\]  

(FIS)

**Refinement in Event-B.** Event-B employs a top-down refinement-based approach to formal development of a system. The development starts from an abstract specification of the system (i.e., an abstract machine) and continues with stepwise unfolding of system properties by introducing new variables and events into the model (i.e., refinements). This type of a refinement is known as a superposition refinement. Moreover, Event-B formal development supports data refinement allowing us to replace some abstract variables with their concrete counterparts. In this case, the invariant of a refined model formally defines the relationship between the abstract and concrete variables; this type of invariants is called a gluing invariant.

To verify correctness of a refinement step, one needs to discharge a number of POs for a refined model. For brevity, here we show only essential ones. The full list of POs can be found in [16].

Let us introduce a shorthand \(H(d,c,v,w)\) that stands for the hypotheses \(A(d,c), I(d,c,v)\) and \(\text{I}(d,c,v,w)\), where \(I\) and \(I'\) are respectively the abstract and the refined invariants, while \(v,w\) are respectively the abstract and concrete variables.

When refining an event, its guard can only be strengthened:

\[
H(d,c,v,w), g'_e(d,c,w) \Rightarrow g_e(d,c,v),
\]  

(GRD)

where \(g_e, g'_e\) are respectively the abstract and concrete guards of the event \(e\).

The simulation proof obligation (SIM) requires to show that the action (i.e., the modelled state transition) of a refined event is not contradictory to its abstract version:

\[
H(d,c,v,w), \text{BA}'_e(d,c,v',w') \Rightarrow \exists v'. \text{BA}_e(d,c,v,v') \land I(d,c,v',w'),
\]  

(SIM)

where \(\text{BA}_e, \text{BA}'_e\) are respectively the abstract and concrete before-after predicates of the same event \(e\), \(v\) and \(w\) are the concrete variable values before and after this event execution.

All the described above proof obligations are automatically generated by the Rodin platform [20] that supports Event-B. Additionally, the tool attempts to automatically prove them. Sometimes it requires user assistance by invoking its interactive prover. However, in general the tool achieves a high level of automation in proving (usually about 85% of POs are proved automatically, e.g., [23,33]).

**Verification via theorem proving.** Additionally, the Event-B formalism allows the developers to formulate theorems either in the model CONTEXT or MACHINE components. In the first case, theorems are logical statements about model static data structures that are provable (derivable) from the model axioms given in the CONTEXT component. In the latter case, these are logical statements about model dynamic properties that follow from the given formal definitions of the model events and invariants.

The theorem proof obligation (THM) indicates that this is a theorem proposed by the developers. Depending whether a theorem is defined in the CONTEXT or MACHINE components, it has a slightly different form. To highlight this difference, we use indexes \(C\) and \(M\) in this paper. The first variant of a proof obligation is defined for a theorem \(T(d,c)\) in the CONTEXT component:

\[
A(d,c) \Rightarrow T(d,c).
\]  

(THM\(_C\))

The second variant is defined for a theorem \(T(d,c,v)\) in the MACHINE component:

\[
A(d,c). I(d,c,v) \Rightarrow T(d,c,v).
\]  

(THM\(_M\))

### 2.2. Safety cases

A safety case is “a structured argument, supported by a body of evidence that provides a convincing and valid case that a system is safe for a given application in a given operating environment” [3,34]. The construction, review and acceptance of safety cases are the valuable steps in safety assurance process of critical software systems. Several standards, e.g., ISO 26262 [1] for the automotive domain, EN 50128 [2] for the railway domain, and the UK Defence Standard 00-56 [3], prescribe production and evaluation of safety
In general, safety cases can be documented either textually or graphically. However, a growing number of industrial companies working with safety–critical systems adopt a graphical notation, namely Goal Structuring Notation (GSN) proposed by Kelly [5], in order to present safety arguments within safety cases [36]. GSN aims at graphical representation of safety case elements as well as the relationships that exist between these elements. The principal building blocks of GSN are shown in Fig. 1. Essentially, a safety case constructed using GSN consists of goals, strategies and solutions. Here goals are propositions in an argument that can be said to be true or false (e.g., claims of requirements to be met by a system). Solutions contain the information extracted from analysis, testing or simulation of a system (i.e., evidence) to show that the goals have been met. Finally, strategies are reasoning steps describing how goals are decomposed and addressed by sub-goals.

Thus, a safety case constructed in GSN presents decomposition of the given safety case goals into sub-goals until they can be supported by the direct evidence (a solution). It also explicitly defines the argument strategies, relied assumptions, the context in which goals are declared, as well as justification for the use of a particular goal or strategy. If the contextual information contains a model, a special GSN symbol called model can be used instead of a regular GSN context element.

The elements of a safety case can be in two types of relationships: “Is solved by” and “In context of”. The former is used between goals, strategies and solutions, while the latter links a goal to a context, a goal to an assumption, a goal to a justification, a strategy to a context, a strategy to an assumption, a strategy to a justification.

To allow for construction of argument patterns, GSN has been extended to represent generalised elements [5,6]. We utilise the following elements from the extended GSN for structural abstraction of our argument patterns: multiplicity and optionality. Multiplicity is a generalised n-ary relationship between the GSN elements, while optionality stands for optional and alternative relationship between the GSN elements. Graphically, the former is represented as a solid ball or a hollow ball on an arrow “Is solved by” shown in Fig. 1, where the label n indicates the cardinality of a relationship, while a hollow ball means zero or one. The latter is depicted as a solid diamond in Fig. 1, where m-of-n denotes a possible number of alternatives. The multiplicity and the optionality relationships can be combined. If a multiplicity symbol is placed in front of the optionality symbol, this stands for a multiplicity over all the options.

There are two extensions for entity abstraction in GSN: (1) uninstantiated entity and (2) undeveloped and uninstantiated entity. The former one specifies that the entity requires to be instantiated, i.e., the “abstract” entity needs to be replaced with a more concrete instance later on. In Fig. 1, the corresponding annotation is depicted as a hollow triangle. It can be used with any GSN element. The latter one indicates that the entity needs both further development and instantiation. In Fig. 1, it is shown as a hollow diamond with a line in the middle. This annotation can be applied to GSN goals and strategies only.

3. Methodology

In this section, we describe our methodology that aims at establishing a link between formal verification of system safety requirements in Event-B and the construction of safety cases.

3.1. General methodology

In this work, we contribute to the process of development, verification and certification of software systems by showing how to proceed from the given system safety requirements to safety cases via formal modelling and verification in Event-B (Fig. 2). Event-B and the accompanying toolset are very suitable for formalising and verifying functional safety requirements. Thus, in this paper we mostly focus on such requirements. To address non-functional system aspects, Event-B should be bridged with other external tools. We will demonstrate this by considering system timing properties (see Section 4.9 for more details).

In general, we distinguish two main processes: (1) representation of formalised system safety requirements in Event-B models, and (2) derivation of safety cases from the associated Event-B specifications. Let us point out that these activities are tightly connected to each other. Accuracy of the system safety requirements formalisation influences whether we are able to construct a safety case sufficient to demonstrate safety of a system. This dependence is highlighted in Fig. 2 as a dashed line. If a formal specification is not good enough, we need to return and improve it.

We connect these two processes via classification of system safety requirements. On the one hand, we propose a specific classification associated with particular ways these requirements can be represented in Event-B. On the other hand, we propose a set of classification-based argument patterns to enable the construction of safety cases from the associated Event-B models. The classification includes separate classes for safety requirements about global and local system properties, the absence of system deadlock, temporal and timing properties, etc. We are going to present this classification in detail in Section 3.2.
In this paper, we leave out of the scope the process of safety requirements elicitation. We assume that the given list of these requirements is completed beforehand by applying well-known hazard analysis techniques such as HAZard and OPerability (HAZOP) analysis, Preliminary Hazard Analysis (PHA), Failure Modes and Effects Analysis (FMEA), etc.

**Incorporating safety requirements into formal models.** Each class of system safety requirements can be treated differently in an Event-B specification (model). In other words, various model expressions based on model elements, e.g., axioms, variables, invariants, events, etc., can be used to formalise a considered safety requirement. Consequently, the argument strategies and resulting evidence in a safety case built based on such a formal model may also vary. Using the defined classification, we provide the reader with the precise guidelines on how to map safety requirements of some class into a particular subset of model elements. Moreover, we define how to construct from these model elements a specific theorem to be verified. Later on, we will show how the verification results (e.g., discharged proof obligations and model checking results) can be used as the evidence in the associated safety cases.

**Constructing safety cases from formal models.** Model-based development in general and development using formal methods in particular typically require additional argumentation about model correctness and well-definedness [39]. In this paper, we address this challenge and provide the corresponding argument pattern as shown in Section 4.1.

Having a well-defined classification of safety requirements benefits both stages of the proposed methodology, i.e., while incorporating safety requirements into formal models and while deriving safety cases from such formal models. To simplify the task of linking the formalised system safety requirements with the safety case to be constructed, we propose a set of classification-based argument patterns (Sections 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9). The patterns have been developed using the corresponding GSN extensions (Fig. 1). Some parts of an argument pattern may remain the same for any instance, while others need to be further instantiated (they are labelled with a specific GSN symbol – a hollow triangle). The text highlighted by braces {} should be replaced by a concrete value.

The generic representation of a classification-based argument pattern is given in Fig. 3. Here, a safety requirement Requirement of some class Class {X} is reflected in the goal GX, where X is a class number (see the next section for the reference). According to the proposed approach, the requirement is verified within a formal model M in Event-B (the model element MX.1).

In order to obtain the evidence that a specific safety requirement is met, different construction techniques might be undertaken. The choice of a particular technique influences the argumentation strategies to be used in each pattern. For example, if a safety requirement can be associated with a model invariant property, the corresponding theorem for each event in the model M is required to be proved. Correspondingly, the proofs of these theorems are attached as the evidence for the constructed safety case.

The formulated properties and theorems associated with a particular requirement can be automatically derived from the given formal model. Nonetheless, to increase clarity of a safety case, any theorem or property whose verification result is provided as a solution of the top goal needs to be referred to in the GSN context element (CX.2 in Fig. 3).

To bridge a semantic gap in the mapping associating an informally specified safety requirement with the corresponding formal expression in Event-B, we need to argue over a correct formalisation of the requirement (SX.2 in Fig. 3). We rely on a joint inspection conducted by domain and formalisation experts (SnX.2) as the evidence that the associated model elements (via the defined mappings) are proper formalisations of the requirement.

**Generating code.** Additionally, the most detailed (concrete) specification obtained during the refinement-based development can be used for code generation. The Rodin platform, Event-B tool support, allows for program code generation utilising a number of plug-ins. One of these plug-ins, EB2ALL [40], automatically generates a target programming language code from an Event-B formal specification. In particular, EB2C allows for generation of C code, EB2CPP supports C++ code generation, using EB2J one can obtain Java code, and using EBC# – C# code. The alternative solution is to use the constructed formal specification for verification of an already existing implementation code. Then, if the code has successfully passed such a verification, the existing safety case derived from the formal specification implies the code safety for the verified safety properties. Nonetheless, in both cases a safety case based on formal analysis cannot be used solely. It requires additional argumentation, for example, over the correctness of the code generation process itself [14,36].

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**Fig. 2.** High-level representation of the overall approach.
3.2. Requirements classification and its mapping into Event-B elements

To classify system safety requirements, we have firstly adopted the taxonomy proposed by Bitsch [41] as presented in our previous work [21]. However, the Bitsch’s approach uses Computational Tree Logic (CTL) to specify the requirements and relies on model checking as a formal verification technique. The differences between the semantics of CTL and Event-B significantly restrict the use of the Bitsch’s classification in the Event-B framework. As a result, we extensively modified the original classification. In this paper, we propose the following classification of system safety requirements, as shown in Fig. 4.

We divide safety requirements (SRs) into eight classes:

- **Class 1**: SRs about global properties are the requirements stipulating the system safety properties that must be always maintained by the modelled system;
- **Class 2**: SRs about local properties are the requirements that reflect the necessity of some property to be true at a specific system state;
- **Class 3**: SRs about control flow are the requirements that define the necessary flow (order) in occurrences of some system events;
- **Class 4**: SRs about the absence of system deadlock are the requirements related to a certain class of control systems where an unexpected stop of the system may lead to a safety-related hazard;
- **Class 5**: SRs about system termination are the requirements related to a certain class of control systems where non-termination of the system in a specific situation may lead to a safety-related hazard;
- **Class 6**: Hierarchical SRs are the requirements that are hierarchically structured to deal with the complexity of the system, i.e., a more general requirement may be decomposed into several more detailed ones;
- **Class 7**: SRs about temporal properties are the requirements that describe the properties related to reachability of specific system states;
- **Class 8**: SRs about timing properties are the requirements that establish timing constraints of a system, for example, of a safety-critical real-time system where the response time is crucial.

The given classes of SRs are represented differently in a formal model. For instance, SRs of Class 1 are modelled as invariants in the MACHINE component, while SRs of Class 2 are modelled by defining a theorem about the required post-state of a specific Event-B model event. However, in some cases requirements of Class 2 can be also formalised as requirements of Class 1 by defining implicative invariants, i.e., invariants that hold in specific system states. The SRs about control flow (Class 3) can be expressed as event-flow
properties (e.g., by using Event-B extension – the graphical Usecase/Flow language [32]). The SRs about the absence of system deadlock (Class 4) are represented as deadlock freedom conditions, while the SRs of Class 5 are modelled as shutdown conditions. In both cases, these conditions are turned into specific model theorems to be proved. The class of hierarchical SRs (Class 6) is expressed within Even-B based on refinement between the corresponding Event-B models. Finally, the associated ProB tool for the Rodin platform [42] allows us to support the SRs of Class 7 by model checking.

Let us note however that the representation of timing properties (Class 8) in the Event-B framework is a challenging task. There are several works dedicated to address this issue [27,43,44,45]. In this paper, we adopt the approach that establishes a link between timing constraints defined in Event-B and verification of real-time properties in Uppaal [27].

Formally, the described above relationships can be defined as a function $F_{SR}$ mapping safety requirements (SRs) into a set of the related model expressions:

\[
SRs \rightarrow P(MExpr),
\]

where $P(T)$ corresponds to a power set on elements of $T$ and $MExpr$ stands for a generalised type for all possible expressions that can be built from the model elements, i.e., model expressions. Here model elements are elements of Event-B models such as axioms, variables, invariants, events, and attributes. $MExpr$ includes such model elements as trivial (basic) expressions. Among other possible expressions of this type are state predicates defining post-conditions and shutdown conditions, event control flow expressions as well as Linear Temporal Logic (LTL) and Timed Computation Tree Logic (TCTL) formulas based on elements of the associated Event-B model.

The defined strict mapping allows us to trace the system safety requirements given in an informal manner into formal specifications in Event-B as well as into the accompanying means for verification, i.e., the Flow and ProB plug-ins and Uppaal. In Fig. 5, we illustrate the steps of evidence construction in our proposed approach. Firstly, we map a safety requirement into a set of model expressions. Secondly, we construct a specific theorem or a set of theorems out of these model expressions, thus essentially defining the semantics of the formalised requirement. Finally, we prove each theorem using the theorem provers of Event-B or perform model checking using, e.g., Event-B extension ProB. As a result, we obtain either a discharged proof obligation or a result of model checking. We include such results into the fragment of a safety case corresponding to the considered safety requirement as the evidence for the proposed argument pattern. In general, such a theorem could be very large in size. Thus, for simplicity, we suggest to divide axioms into groups, where each group consists of axioms that use shared constants and sets. In other words, each group of axioms is independent from each other. Consequently, we define theorems for all groups of independent axioms (the strategy S1.2) as shown below:

\[
\text{thm}_{axm}(i) : A(d,c) \rightarrow \exists d, c \cdot A_1(d,c) \land \ldots \land A_K(d,c),
\]

where $i$ stands for $i$-th group of axioms such that $i \in 1, \ldots, K$ and $K$ is the number of independent groups of axioms. The number of axioms in a group is represented by $N$. The generated proof obligation (Sn1.1) shown in Fig. 6 is an instance of the (THM$_i$) proof obligation given in Section 2.1.

In order to instantiate this pattern for each model in the development,

- a formal development that consists of a chain of refinements in Event-B should be defined in a GSN model element;
- a formal model $M$, for which a particular fragment of the safety case is constructed, should be referred to in a GSN model element;
- theorems about the defined groups of independent axioms should be formulated using the Event-B formal language and referred to in GSN context elements;
- the proof obligations of the type $\text{THM}_i$ discharged by the Rodin platform should be included as solutions of the goal ensuring consistency of model axioms.

We illustrate instantiation of this fragment of the safety case on a case study – a steam boiler control system [22]. This is a safety-critical control system that produces steam and adjusts the quantity of water in the steam boiler chamber to maintain it within the predefined safety boundaries. The situations when the water level is too low or too high might result in loss of personnel life, signif-
critical equipment damage (the steam boiler itself or the turbine placed in front of it), or damage to the environment.

Our Event-B development of the steam boiler system presented in [23] consists of an abstract specification and its four refinements. Due to a significant size of the system safety case, here we show only a part of the instantiated pattern for demonstrating well-definedness of a formal development. Fig. 7 presents the resulting fragment of the safety case concerning the first refinement model (MACHINE M1 and the associated CONTEXT C1).

To apply the well-definedness argument pattern, we have to formally demonstrate axiom consistency in the CONTEXT C1. We argue over axiom consistency by defining two groups of axioms. The first group includes axioms defining generic parameters of the system, e.g., the constants associated with the criticality of the water level, which is based on the pre-defined safety boundaries. The second group consists of the axioms defining the abstract function Stable needed to model the failure stability property. Here stability means that, once a failure occurred, the value of the variable representing this failure remains unchanged until the whole system is restarted. These groups are independent because they refer to distinct Event-B constants and sets. The corresponding theorems thm_axm1 and thm_axm2 are shown below. The first theorem verifies that the parameters of the steam boiler are introduced in the model correctly (i.e., their definitions are non-contradictory):

$$\text{thm\_axm1} : \exists N1, N2, M1, M2, C, Wl\text{\_critical}$$

$$\cdot N1 \in N1 \land N2 \in N1 \land$$

$$M1 \in N1 \land M2 \in N1 \land$$

$$C \in N1 \land Wl\text{\_critical} \in N \times N \rightarrow BOOL \land$$

$$0 < M1 \land M1 < N1 \land N1 < N2 \land$$

$$N2 < M2 \land M2 < C \land$$

$$((x < M1 \lor y > M2))$$

$$\iff$$

$$WL\text{\_critical}(x \rightarrow y) = TRUE)) \land$$

$$((x < M1 \lor y > M2))$$

$$\iff$$

$$WL\text{\_critical}(x \rightarrow y) = FALSE)).$$

Similarly, the second theorem verifies that the group of axioms introduced to define a function about the failure stability is consistent:

$$\text{thm\_axm2} : \exists Stable \cdot Stable \in BOOL \times BOOL \rightarrow BOOL \land$$

$$((\forall x, y \cdot x \in BOOL \land y \in BOOL \Rightarrow$$

$$Stable(x \rightarrow y) = TRUE \iff (x = TRUE \Rightarrow y = TRUE))).$$

The obtained proofs of these theorems are included in the safety case as the solutions Sn1.1 and Sn1.2 correspondingly.

In the remaining part of Section 4, we introduce the argument patterns that correspond to each class of safety requirements proposed in Section 3.2. It is not necessarily the case that the final safety case of the modelled system will include SRs of all the classes. Moreover, it is very common for the Event-B practitioners to limit the requirements model representation to invariants, theorems and operation refinement [46–48]. However, to achieve a strong safety case, the developers need to provide the evidence that all the safety requirements listed in the requirements document hold. The proposed argument patterns cover a broader range of system safety requirements, including also those that specify temporal and timing properties which cannot be formalised in Event-B directly.

4.2. Argument pattern for SRs about global properties (Class 1)

In this section, we propose an argument pattern for the safety requirements stipulating global safety properties, i.e., properties that must be maintained by the modelled system during its operation (Fig. 8).

We assume that there is a model M, which is a part of the formal development of a system in Event-B, where a safety requirement of
Class 1 is verified to hold (M2.1.1). In addition, we assume that the model invariant \( I(d, c, v) \) contains the conjuncts \( \text{safety}_1, \ldots, \text{safety}_N \), where \( N \) is the number of safety invariants, which together represent the proper formalisation of the considered safety requirement (A2.1.1).\(^2\) We show in Fig. 3 how such an assumption can be substantiated by arguing over formalisation of the requirement (the strategy SX.2 in Fig. 3). Then, for each safety invariant \( \text{safety}_i \), \( i \in 1, \ldots, N \), the event \( \text{Event}_k \), \( k \in 1, \ldots, K \), where \( K \) is the number of all model events, represents some event for which this invariant must hold.

We build the evidence for the safety case in the way illustrated in Fig. 5. Thus, for each model expression, in this case, an invariant, formalising the safety requirement, we construct a separate branch of the safety case. Then, a separate theorem is defined for each event where a particular invariant should hold. In other words, we define a group of theorems, one per each event. The number of model events influences the number of branches into which the goal G2.1.1 is split. In a special case when the set of variables referred to in an invariant is mutually exclusive with the set of variables modified by an event, such an event can be excluded from the list of events because the theorem generated for such an event is trivially true.

According to our approach, the generic mapping function \( F_M \) is of the form \( SRs \rightarrow P(MExpr) \). In general case, for each requirement of this class the function returns a set of invariants \( \{\text{safety}_1, \ldots, \text{safety}_N\} \) that can be represented as a conjunction. Due to this fact, each such an invariant can be verified independently. The theorem for verification that the safety invariant \( \text{safety}_i \) (denoted by \( I(d, c, v') \)) holds for the event \( \text{Event}_k \) is as follows:

\[
A(d, c), I(d, c, v), g_{\text{Event}_k}(d, c, v), BA_{\text{Event}_k}(d, c, v, v') \vdash I(d, c, v').
\]

\(^2\) Such an assumption can be substantiated by arguing over formalisation of the requirements as demonstrated in Fig. 3 (the strategy SX.2). Such substantiation is applicable to all the classification-based argument patterns and their instances.
The Rodin platform allows us to prove this theorem using the integrated theorem provers and explicitly support the safety case to be built with the discharged proof obligations of the type INV for each event where the safety invariant has been verified to hold.

The key elements of the pattern to be instantiated are as follows:

- a requirement Requirement should be replaced with a particular safety requirement;
- a formal model M should be referred to in a GSN model element;
- the concrete mapping between the requirement and the corresponding model invariants should be provided, while the invariants safety1, ..., safetyN formalising the requirement from this mapping should be referred to in GSN context elements;
- the proof obligations of the type INV discharged by the Rodin platform should be included in the safety case as the respective solutions.

Let us consider instantiation of the proposed pattern by an example – a sluice gate control system [24]. The system is a sluice connecting areas with dramatically different pressures. The purpose of the system is to adjust the pressure in the sluice area connecting outside and inside areas with the sluice area. To guarantee safety, a door may define a (safety) mode which the system enters after the execution of some transition. In terms of Event-B, the particular system states we are interested in are usually associated with some desired post-states of specific model events.

Safety requirements of Class 2 describe local properties, i.e., the properties that need to be true at specific system states. For example, in case of a control system relying on the notion of operational modes, a safety requirement of Class 2 may define a (safety) mode which the system enters after the execution of some transition. In terms of Event-B, the particular system states we are interested in are usually associated with some desired post-states of specific model events.

Fig. 10 shows the argument pattern for justification of a safety requirement of Class 2. As for Class 1, the key argumentation strategy here (S2.2.1) is defined by the steps of evidence construction illustrated in Fig. 5. However, in contrast to the invariant theorems established and proved for each event in the model, the theorem formalising the safety requirement of Class 2 is formulated and proved only once for the whole model M.
As mentioned above, local properties are usually expressed in Event-B in terms of post-states of specific model events. This suggests the mapping function \( F_M \) for Class 2 to be of the form:

\[
\text{Requirement} 
\rightarrow \{ (e_1, q_1), \ldots, (e_k, q_k) \},
\]

where the events \( e_1, \ldots, e_k \) and the state predicates \( q_1, \ldots, q_k \) are model expressions based on which the corresponding theorems are constructed. The number of such theorems reflects the number of branches of a safety case for the goal \( G2.2 \) (Fig. 10). Specifically, we can verify a safety requirement of Class 2 by proving the following theorem for each pair \( (e_i, q_j) \), where \( i \in 1, \ldots, K \) and \( j \in 1, \ldots, S \):

\[
\text{thm}_{ap} : \quad A(d, c) \land I(d, c, v) \implies \forall v' : v' \in \text{after}(e_i) \implies q_j(v').
\]

Here \( \text{after}(e_i) \) is the set of all possible post-states of the event \( e_i \) as defined in Section 2.1.

This argument pattern (Fig. 10) can be instantiated as follows:

- a requirement \( \text{Requirement} \) should be replaced with a particular safety requirement;
- a formal model \( M \) should be referred to in a GSN model element;
- the concrete mapping between the requirement and event-post-condition pairs should be supplied, while the theorems \( \text{thm}_{ap} \) obtained from this mapping should be referred to in GSN context elements;
- the proof obligations of the type \( \text{THM}_M \) discharged by the Rodin platform should be included in the safety case as the evidence supporting that the top-level claim (i.e., \( G2.2 \)) holds.

In order to demonstrate application of this pattern, let us introduce another case study – attitude and orbit control system (AOCs) [25]. The AOCs is a typical layered control system. The main function of this system is to control the attitude and the orbit of a satellite. Since the orientation of a satellite may change due to disturbances of the environment, the attitude needs to be continuously monitored and adjusted. At the top layer of the system there is a mode manager (MM). The transitions between modes can be performed either to fulfill the predefined mission of the satellite (forward transitions) or to perform error recovery (backward transitions). Correspondingly, the MM component might be in either stable, increasing (i.e., in forward transition) or decreasing (i.e., in backward transition) state. As an example, let us consider the safety requirement:

**SR-cl2:** When a mode transition is completed, the state of the MM shall be stable.

To verify this property on model events and variables, we need to prove that the corresponding condition \( q \), namely

\[
\text{last} \_ \text{mode} = \text{prev} \_ \text{target} \land \text{next} \_ \text{target} = \text{prev} \_ \text{target},
\]

holds after the execution of the event \( \text{Mode} \_ \text{Reached} \). Here \( \text{prev} \_ \text{target} \) is the previous mode that a component was in transition to, \( \text{last} \_ \text{mode} \) is the last successfully reached mode, and \( \text{next} \_ \text{target} \) is the target mode that a component is currently in transition to. The event is enabled only when there is no critical error in the system, i.e., when the condition \( \text{error} = \text{No} \_ \text{Error} \) holds.

We represent the mapping of the shown safety requirement on Event-B as \( F_M \) such that \( \text{SR-cl2} \mapsto [(\text{Mode} \_ \text{Reached}, q)] \). According to \( \text{thm}_{ap} \) and the definition of \( \text{after}(e) \) given in Section 2.1, we can construct the theorem to be verified as follows:

\[
\text{thm}_{cl2} \_ \text{ex} : \quad \forall \text{last} \_ \text{mode}', \text{prev} \_ \text{targ}', \text{next} \_ \text{targ}' .
\]

\[
(\exists \text{next} \_ \text{targ}, \text{error}, \text{prev} \_ \text{targ} .
\]

\[
(\text{next} \_ \text{targ} \neq \text{prev} \_ \text{targ} \land \text{error} = \text{No} \_ \text{Error}) \land
\]

\[
(\text{last} \_ \text{mode}' = \text{next} \_ \text{targ} \land \text{prev} \_ \text{targ}' = \text{next} \_ \text{targ} \land
\]

\[
\text{next} \_ \text{targ}' = \text{next} \_ \text{targ} \implies \text{last} \_ \text{mode}' = \text{next} \_ \text{targ} \land \text{prev} \_ \text{targ}' = \text{next} \_ \text{targ}.
\]

Here, for simplicity, we omit showing types of the involved variables. The corresponding instance of the argument pattern is illustrated in Fig. 11.

4.4. Argument pattern for SRs about control flow (Class 3)

In this section, we propose an argument pattern for the requirements that define the flow in occurrences of some system events, i.e., safety requirements about control flow. For instance, this class may include certain requirements that define fault-tolerance procedures. Since fault detection, isolation and recovery actions are strictly ordered, we also need to preserve this order in a formal model of the system.

Formally, the ordering between system events can be expressed as a particular relationship amongst possible pre- and post-states
of the corresponding model events. We consider three types of relationships proposed by Iliasov [32]: enabling (ena), disabling (dis) and possibly enabling (fis). In detail, enabling relationship between two events means that, when one event occurs, it is always true that the other one may occur next (i.e., the set of pre-states of the second event is included in the set of post-states of the first event). An event disables another event if the guard of the second event is always false after the first event occurs (i.e., the set of pre-states of the second event is excluded from the set of post-states of the first event). Finally, an event possibly enables another event if, after its occurrence, the guard of the second event is potentially enabled (i.e., there is a non-empty intersection of the set of pre-states of the second event with the set of post-states of the first event).

Let $e_0$ and $e_n$ be some events. Then, according to the usecase/flow approach [32], the proof obligations that support the relationships between these events can be defined as follows:

$$\begin{align*}
\text{ena}_{e_m} & \iff \text{after}(e_m) \subseteq \text{before}(e_n) \\
& \iff \forall v, v'. \ [v] \land g_{e_m}(v) \land BA_{e_m}(v, v') \Rightarrow g_{e_n}(v').
\end{align*}$$

(FENA)

$$\begin{align*}
\text{dis}_{e_m} & \iff \text{after}(e_m) \cap \text{before}(e_n) = \emptyset \\
& \iff \forall v, v'. \ [v] \land g_{e_m}(v) \land BA_{e_m}(v, v') \Rightarrow \neg g_{e_n}(v').
\end{align*}$$

(FDIS)

$$\begin{align*}
\text{fis}_{e_m} & \iff \text{after}(e_m) \land \text{before}(e_n) \\
& \iff \exists v, v'. \ [v] \land g_{e_m}(v) \land BA_{e_m}(v, v') \land g_{e_n}(v').
\end{align*}$$

(FFIS)

The flow approach and its supporting plug-in for the Rodin platform, called Usecase/Flow plug-in [49], allows us to derive these proof obligations automatically.

The argument pattern shown in Fig. 12 pertains to the required events order (C2.3.2) which is proved to be preserved by the respective events of a model $M$. As explained above, each event $E_{\text{Event}_i}$ can be either enabled (ena), or disabled (dis), or possibly enabled (fis) by some other event $E_{\text{Event}_j}$. This suggests that the mapping function $F_{\text{Map}}$ is of the form:

$$\text{Requirement} \rightarrow \{(E_{\text{Event}_i}, \text{ena}, E_{\text{Event}_j}), (E_{\text{Event}_i}, \text{dis}, E_{\text{Event}_j}), (E_{\text{Event}_i}, \text{fis}, E_{\text{Event}_j}), \ldots\}.$$

The corresponding theorem is constructed according to the definition of either (FENA), or (FDIS), or (FFIS). Then, the discharged proof obligations for each such a pair of events are provided as the evidence in a safety case, e.g., Sn2.3.1 in Fig. 12.

The instantiation of the proposed argument pattern can be achieved by preserving a number of the following steps:

- a requirement Requirement should be replaced with a particular safety requirement;
- a formal model $M$ should be referred to in a GSN model element;
- the concrete mapping between the requirement and the corresponding pairs of events and relationships between them should be provided, while the required events order based on this mapping should be referred to in a GSN context element;
- a separate goal for each pair should be introduced in the safety case;
- each goal that claims the enabling relationship between events should be supported by the proof obligation of the type FENA in a GSN solution element;
- each goal that claims the disabling relationship between events should be supported by the proof obligation of the type FDIS in a GSN solution element;
- each goal that claims the possibly enabling relationship between events should be supported by the proof obligation of the type FFIS in a GSN solution element.

In the already introduced case study AOCs (Section 4.3), there is a set of requirements regulating the order of actions to take place in the system control flow. These requirements define the desired rules of transitions between modes, e.g.,

**SR-cl3**: The system shall perform its (normal or failure handling) operation only when there are no currently running transitions between modes at any level.

This means that once a transition is initiated either by the high-level mode manager or lower level managers, it has to be completed before system operation continues.

As an example, we consider a formalisation of the requirement SR-cl3 at the most abstract level, i.e., the MACHINE MM.Abs_M and the CONTEXT MM.Abs.C where the essential behaviour of the high-level mode manager is introduced.

The required events order (C2.3.2) is depicted by the usecase/flow diagram in Fig. 13. This flow diagram can be seen as a use case scenario specification attached to the MACHINE MM.Abs_M. The presented flow diagram is drawn in the graphical editor for the Usecase/Flow plug-in for the Rodin platform. While defining the desired relationships between events using this editor, the
corresponding proof obligations are generated automatically by the Rodin platform.

In terms of the usecase/flow approach, the requirement SR-cl3 states that the event Advance\_partial enables the event Advance\_complete and disables operation events Normal\_Operation and Failure\_Operation. In its turn, the event Advance\_complete disables the event Advance\_partial and enables system (normal or failure handling) operation events. Then, the mapping function $F_M$ is instantiated as follows:

$$SR\text{-}cl3 \mapsto \{(\text{Advance\_partial, ena}, \text{Advance\_complete}), \quad (\text{Advance\_partial, dis}, \text{Normal\_Operation}), \quad (\text{Advance\_partial, dis}, \text{Failure\_Operation}), \quad (\text{Advance\_complete, dis}, \text{Advance\_partial}), \quad (\text{Advance\_complete, ena}, \text{Normal\_Operation}), \quad (\text{Advance\_complete, ena}, \text{Failure\_Operation})\}.$$  

The instance of the argument pattern for the safety requirement SR-cl3 is shown in Fig. 14.

4.5. Argument pattern for SRs about the absence of system deadlock (Class 4)

In this section, we propose an argument pattern for the safety requirements stipulating the absence of the unexpected stop of the system (Fig. 15). We formalise requirements of Class 4 within an Event-B model $M$ as the deadlock freedom theorem. Similarly to the SRs of Class 2, this theorem has to be proved only once for the whole model $M$. The theorem is reflected in the argument strategy that is used to develop the main goal of the pattern (S2.4.1 in Fig. 15).

Formally, the deadlock freedom theorem is formulated as the disjunction of guards of all model events $g_1(d, c, v) \lor \ldots \lor g_K(d, c, v)$, where $K$ is the total number of model events:

$$\text{thm\_dlf} : \quad A(d, c), I(d, c, v) \vdash g_1(d, c, v) \lor \ldots \lor g_K(d, c, v).$$

The corresponding mapping function $F_M$ for this argument pattern is defined as $\text{Requirement} \mapsto \{\text{event}_1, \ldots, \text{event}_n\}$. Then, the instance of the (THM\_dlf) proof obligation given in Section 2.1 provides the evidence for the safety case (Sn2.4.1 in Fig. 15).

The argument pattern presented in Fig. 15 can be instantiated as follows:

- a requirement Requirement should be replaced with a particular safety requirement;
- a formal model $M$ should be referred to in a GSN model element;
- the concrete mapping between the requirement and the corresponding model events should be supplied, while the theorem $\text{thm\_dlf}$ formalising the requirement from this mapping should be referred to in a GSN context element;
- the proof obligation of the type THM\_dlf discharged by the Rodin platform should be included in the safety case as the evidence supporting that the top-level claim (i.e., G2.4 in Fig. 15) holds.
We illustrate the instantiation of this argument pattern by a simple example presented by Abrial in Chapter 2 of [16]. The considered system performs controlling cars on a bridge. The bridge connects the mainland with an island. Cars can always either enter the compound or leave it. Therefore, the absence of the system deadlock should be guaranteed, i.e.,

**SR-cl4**: Once started, the system should work for ever.

The semantics of Event-B allows us to chose the most abstract specification to argue over the deadlock freedom of a system. According to the notion of the relative deadlock freedom, which is a part of the Event-B semantics, new deadlocks cannot be introduced in a refinement step.\(^3\) As a consequence, once the model is proved to be deadlock free, no new refinement step can introduce a deadlock.

The abstract model of the system has three events: Initialisation, ML\(_\text{out}\) and ML\(_\text{in}\). Thus, the concrete mapping function \(FM\) is as follows:

\[
\text{SR-cl4} \rightarrow \{\text{Initialisation}, \text{ML\textunderscore out}, \text{ML\textunderscore in}\}.
\]

Here \(\text{ML\textunderscore out}\) models leaving the mainland, while \(\text{ML\textunderscore in}\) models entering the mainland. The former event has the guard \(n < d\), where \(n\) is a number of cars on the bridge and \(d\) is a maximum number of cars that can enter the bridge. The latter event is guarded by the condition \(n > 0\), which allows this event to be enabled only when some car is on the island or the bridge. Therefore, the corresponding deadlock freedom theorem \(\text{thm\textunderscore cl4}\) can be defined as follows:

\[
\text{thm\textunderscore cl4}\colon n > 0 \lor n < d.
\]

The event Initialisation does not have a guard and therefore is not reflected in the theorem. The instantiated fragment of the safety case for this example is shown in Fig. 16.

The details on the considered formal development in Event-B (Controlling cars on a bridge) as well as the derived proof obligation of the deadlock freedom can be found in [16,50].

4.6. Argument pattern for SRs about system termination (Class 5)

In contrast to Class 4, Class 5 contains the safety requirements stipulating the system termination in particular cases. For instance,
it corresponds to failsafe systems (i.e., systems which need to be put into a safe but non-operational state to prevent an occurrence of a hazard). Despite the fact that the argument pattern is quite similar to the one about the absence of system deadlock, this class of safety requirements can be considered as essentially opposite to the previous one. Here the requirements define the conditions when the system must terminate. More specifically, the system is required to have a deadlock either (1) in a specific state of the model M, i.e., after the execution of some event $e_i$ (where $i \in 1, \ldots, K$ and $K$ is the total number of model events), or (2) once a shutdown condition ($\text{shutdown}_{\text{cond}}$) is satisfied:

1. $\text{after}(e_i) \land \text{before}(E) = 0$.
2. $\text{shutdown}_{\text{cond}} \land \text{before}(E) = 0$.

where $\text{shutdown}_{\text{cond}}$ is a predicate formalising a condition when the system terminates and $\text{before}(E)$ is defined as a union of pre-states of all the model events:

$$\text{before}(E) = \bigcup_{e_i \in E} \text{before}(e_i).$$

Correspondingly, the mapping function $F_M$ for Class 5 can be either of the form

1. $\text{Requirement} \mapsto \{e_i, e_1, \ldots, e_K\}$, or
2. $\text{Requirement} \mapsto \{\text{state predicate}, e_1, \ldots, e_K\}$,

where state predicate is a formally defined shutdown condition.

Then, for the first case, the theorem about a shutdown condition has the following form:

$$\text{thm}_{\text{shd}} : \begin{aligned} \text{before}(E) \land (\text{after}(e_i) \Rightarrow \neg \text{before}(E)), \end{aligned}$$

while, for the second case, it is defined as:

$$\text{thm}_{\text{shd}} : \begin{aligned} \text{before}(E) \land (\text{shutdown}_{\text{cond}} \Rightarrow \neg \text{before}(E)), \end{aligned}$$

The argument pattern presented in Fig. 17 can be instantiated as follows:

- a requirement $\text{Requirement}$ should be replaced with a particular safety requirement;
- a formal model $M$ should be referred to in a GSN model element;
- the concrete mapping between the requirement and the corresponding model events (and state predicates) should be provided, while the theorem $\text{thm}_{\text{shd}}$ formalising the require-

ment from this mapping should be referred to in a GSN context element;
- the proof obligation of the type $\text{THM}_M$ discharged by the Rodin platform should be included in the safety case as the evidence supporting that the top-level claim (i.e., $\text{G2.5}$) holds.

To show an example of the pattern instantiation, let us consider the sluice gate control system [24] described in detail in Section 4.2. This system is a failsafe system. To handle critical failures, it is required to raise an alarm and terminate:

**SR-cl5**: When a critical failure is detected, an alarm shall be raised and the system shall be stopped.

Thus, we need to assure that our model also terminates after the execution of the event which sets the alarm on (i.e., the event SafeStop in the model). This suggests the concrete instance of the mapping function $F_M$ to be of the form:

**SR-cl5** $\mapsto$ ($\text{SafeStop. Environment. Detection. door1, \ldots, close2. closed2}$).

Then, the corresponding theorem $\text{thm}_{\text{cl5-ex}}$, which formalises the safety requirement $\text{SR-cl5}$, can be formulated as follows:

$$\begin{aligned} &\text{thm}_{\text{cl5-ex}} : \begin{aligned} \forall \text{flag}. \text{ Stop}. \end{aligned} \\ &\begin{aligned} &\text{flag} = \langle \text{door1 fail. door2 fail. pressure fail. Stop}. \\ &\text{flag} = \langle \text{CONT} \land (\text{door1 fail} = \text{TRUE} \lor \text{door2 fail} = \text{TRUE} \lor \text{pressure fail} = \text{TRUE}) \land \text{Stop} = \neg \text{flag} \rangle \end{aligned} \end{aligned}$$

$$\Rightarrow$$

$$\neg \langle \text{before}(\text{Environment}) \lor \text{before}(\text{Detection. door1}) \lor \ldots \lor \text{before}(\text{close2}) \lor \text{before}(\text{closed2}) \rangle,$$

where the variable $\text{flag}$ indicates the current phase of the sluice gate controller, while the variables $\text{door1 fail. door2 fail. pressure fail}$ stand for failures of the system components (the doors and the pressure pump respectively). The variable $\text{Stop}$ models an alarm and a signal to stop the physical operation of the system components. Finally, $\text{Environment. Detection. door1, \ldots, close2}$ are model events. The corresponding instance of the argument pattern is given in Fig. 18.

4.7. Argument pattern for Hierarchical SRs (Class 6)

Sometimes a whole requirements document or some particular requirements of a system may be structured in a hierarchical way. For example, a general safety requirement may stipulate actions to be taken in the case of a system failure, while more specific safety requirements elaborate on the general requirement by defining how the failures of different system components may contribute to such a failure of the system as well as regulate the actions to mitigate these failures. Often, the numbering of requirements may indicate such intended hierarchical relationships. A more general requirement can be numbered $\text{REQ X}$, while its more specific versions – $\text{REQ X.1, REQ X.2}$, etc. In our classification, we call such requirements Hierarchical SRs.

The class of Hierarchical SRs (Class 6) differs from the previously described classes since it involves several, possibly quite different yet hierarchically linked requirements. The involved individual requirements (a more general (higher-level) requirement and its more detailed (lower-level) counterparts) can be shown to hold separately in different models of the system development in Event-B, by instantiating suitable argument patterns from the described classes 1–5. To ensure the consistency of their hierarchical link, an
additional fragment of a safety case is needed. This fragment illustrates that the formalisation of the involved requirements is consistent, even if it is done in separate models of the Event-B formal development. To address the class of hierarchical requirements, in this section we propose an argument pattern that facilitates the task of construction of such an additional fragment of a safety case. Nevertheless, these requirements can be shown to hold separately, as proposed by our other argument patterns, without instantiating the argument pattern for Class 6. All other classes of safety requirements and the associated argument patterns are orthogonal and can be used independently from each other.

Since the main property of the employed refinement approach is the preservation of consistency between the models, it is sufficient for us to show that the involved models are valid refinements of one another. In Event-B, to guarantee consistency of model transformations, we need to show that the concrete events refine their abstract versions by discharging the corresponding proof obligations to verify guard strengthening (GRD) and action simulation (SIM), as given in Section 2.1. This procedure may involve the whole set of the refined events. However, to simplify the construction of the corresponding fragment of a safety case, we limit the number of events by choosing only those events that are affected by the requirements under consideration. To achieve this, we rely on the given mappings for higher-level and lower-level requirements, returning the sets of the involved model expressions $M_{\text{abs}}(\text{Expr}_{\text{h}})$ and $M_{\text{concr}}(\text{Expr}_{\text{l}})$. Making a step further, we can always obtain the set of affected model events:

$$M_{\text{concr}}(\text{Expr}_{\text{l}}) \Rightarrow \{\text{Event}_{\text{h}1}, \ldots, \text{Event}_{\text{hn}}\}.$$  

As a result, we attach proofs only for those events from $\{\text{Event}_{\text{h}1}, \ldots, \text{Event}_{\text{hn}}\}$ that refine some events from $\{\text{Event}_{\text{l}1}, \ldots, \text{Event}_{\text{ln}}\}$.

Each higher-level requirement may be linked with a set of more detailed lower-level requirements in the requirements document. Nevertheless, to simplify the task, let us consider the case where there is only one such lower-level requirement. If there are more than one such a requirement, one could reiterate the proposed approach by building a separate fragment of a safety case for each pair of linked requirements.

In Fig. 19, Higher-level req. stands for some higher-level requirement, while Lower-level req. is a requirement that is a more detailed version of the higher-level one. The higher-level requirement is mapped onto a formal model $M_{\text{abs}}$, and the lower-level requirement is mapped onto a formal model $M_{\text{concr}}$ (where $M_{\text{concr}}$ is a refinement of $M_{\text{abs}}$) using one of the mapping functions defined for the classes 1–5.

Following the procedure described above, we can associate Higher-level req. with the set of affected events $\{\text{Event}_{\text{h}1}, \ldots, \text{Event}_{\text{hn}}\}$. Similarly, Lower-level req. is associated with its own set of affected events $\{\text{Event}_{\text{l}1}, \ldots, \text{Event}_{\text{ln}}\}$.

For each pair of events $\text{Event}_{\text{h}}$ and $\text{Event}_{\text{l}}$ from the obtained sets, the following two generated proof obligations (GRD) and (SIM) are needed to be proved to establish correctness of model refinement (Section 2.1):

$$H(d, c, v, w), g^e_{\text{Event}}(d, c, v) \vdash g_{\text{Event}}(d, c, v).$$

$$H(d, c, v, w), g^e_{\text{Event}}(d, c, v), B_{\text{Event}}(d, c, w, w') \vdash \exists v'. B_{\text{Event}}(d, c, v, v') \land f(d, c, v').$$

The established proofs of the types GRD and SIM serve as solutions in our pattern, $\text{Sn2.6.1}$ and $\text{Sn2.6.2}$ in Fig. 19 respectively.

The instantiation of the pattern proceeds as shown below:

- requirements Higher-level req. and Lower-level req. should be replaced with specific requirements;
- a more abstract formal model $M_{\text{abs}}$ and a more concrete formal model $M_{\text{concr}}$ should be referred to in a GSN model element;
- the pairs of the associated events of the respective abstract and concrete system models should be referred to in GSN context elements;
- the proof obligations of the types GRD and SIM discharged by the Rodin platform should be included in the safety case as solutions.

Moreover, there can be several hierarchical levels of requirements specification. To cope with this case, we propose to instantiate patterns for each such a level separately.

To illustrate the construction of a safety case fragment for this class of requirements, we refer to the sluice gate control system [24] described in Sections 4.2 and 4.6. Some safety requirements of this system are hierarchically structured. Thus, there is a more generic safety requirement $\text{SR-cl6-higher-level}$:

**SR-cl6-higher-level**: The system shall be able to handle a critical failure by either initiating a shutdown or a recovery procedure stipulating that some actions should take place in order to tolerate any critical failure. However, it does not define the precise procedures associated with this failure handling. In contrast, there is a more detailed counterpart $\text{SR-cl6-lower-level}$ of the requirement $\text{SR-cl6-higher-level}$ (it was presented in the previous section as the requirement $\text{SR-cl5}$). It regulates precisely that an alarm should be raised and the system should stop its operation (the system should terminate):
SR-cl6-lower-level: When a critical failure is detected, an alarm shall be raised and the system shall be stopped.

These safety requirements are shown to hold in different models of the system development. The requirement SR-cl6-higher-level is formalised as two invariants at the most abstract level of the formal specification in Event-B, the MACHINE m1, while the requirement SR-cl6-lower-level is formalised as a theorem in the MACHINE SR-cl6-lower-level specification in Event-B, the MACHINE formalised as two invariants at the most abstract level of the formal

The handling of critical failures is non-deterministically modelled in the abstract Event-B model (Fig. 20). The local variable res is of the type BOOL and can be either TRUE or FALSE. It means that, if a successful error handling procedure that does not lead to the system termination has been performed, both variables standing for a critical failure (Failure) and for the system shutdown (Stop) are assigned the values FALSE and the system continues its operation. Otherwise, they are assigned the values TRUE leading to the system termination.

The fragment of a safety case for the safety requirement SR-cl6-higher-level can be constructed preserving the instructions determined in Section 4.2, while the fragment of a safety case for the requirement SR-cl6-lower-level can be found in Section 4.6.

Now let us focus on ensuring the hierarchical link between these requirements by instantiating the argument pattern for Class 6. Following the proposed approach, we define a set of the affected model events for the higher-level safety requirement: {Environment, Detection, ErrorHandling, Prediction, NormalOperation}, and for the lower-level safety requirement: {Environment, Detection_NoFault, Detection_Fault, SafeStop, Prediction, Normal_Skip}. For simplicity, here we consider only one pair of events ErrorHandling and SafeStop shown in Fig. 20.

In the Event-B development of the sluice gate system, the non-determinism modelled by the local variable res is eliminated via introduction of a specific situation leading to the system shutdown. All other fault tolerance procedures are left out of the scope of the presented development.

Additionally to the introduction of the deterministic procedures for error handling, the variable Failure is data refined in the first refinement m1. Now, the system failure may occur either if the component door1 fails (door1_fail = TRUE), or door2 fails (door2_= fail = TRUE), or the pressure pump fails (pressure_fail = TRUE). This relationship between the old abstract variable and new concrete ones is defined by the corresponding gluing invariant.

The corresponding instance of the argument pattern is presented in Fig. 21. To ensure that the requirement SR-cl6-lower-level is the proper elaboration of the requirement SR-cl6-higher-level (the goal G2.6 in Fig. 21), we argue over the abstract event ErrorHandling and the refined event SafeStop. We show that the guard grd2 is strengthened in the refinement (the discharged proof obligation (GRD)) and the action act2 is not contradictory to the abstract version (SIM). The corresponding proof obligations are shown in Fig. 22.

4.8. Argument pattern for SRs about temporal properties (Class 7)

So far, we have considered the argument patterns of safety requirements classes where the evidence that the top goal of the pattern holds is constructed based on the proof obligations gener-
ated by the Rodin platform. Not all types of safety requirements can be formally demonstrated in this way, however. In particular, the Event-B framework lacks direct support of temporal system properties such as reachability, liveness, existence, etc. Nevertheless, the Rodin platform has an accompanying plug-in, called ProB [51], which allows for model checking of temporal properties. Therefore, in this section, we propose an argument pattern for the class of safety requirements that can be expressed as temporal properties. The pattern is graphically shown in Fig. 23. Here property \( \{ \text{property}_i \} \) stands for some temporal property to be verified, for \( i \in 1, \ldots, N \), where \( N \) is the number of temporal properties of the system.

The property to be verified should be formulated as an LTL formula in the LTL Model Checking wizard of the ProB plug-in for some particular model \( M \). This suggests the mapping function \( F_M \) for Class 7 to be of the form

\[
\text{Requirement} \rightarrow \{ \text{LTL formula} \}.
\]

Each such a temporal property should be well-defined according to restrictions imposed on LTL in ProB. The tool can generate three possible results: (1) the given LTL formula is true for all valid paths (no counter-example has been found, all nodes have been visited); (2) there is a path that does not satisfy the formula (a counter-example has been found and it is shown in a separate view); and (3) no counter-example has been found, but the temporal property in question cannot be guaranteed because the state space was not fully explored.

To instantiate this pattern, one needs to proceed as follows:

- a requirement Requirement should be replaced with a particular safety requirement;
- a formal model \( M \) should be referred to in a GSN model element;
- the concrete mapping between the requirement and the corresponding LTL formula should be supplied, while each temporal property \( \text{property}_i \) from this mapping should be referred to in a GSN context element;
- model checking results on an instantiated property that have been generated by ProB should be included as the evidence that this property is satisfied.

There are several alternative ways to reason over temporal properties in Event-B [52–55]. The most recent of them is that of Hoang and Abrial [54]. The authors propose a set of proof rules

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Fig. 21. The pattern instantiation example.

Fig. 22. The proof obligations of the types GRD and SIM.

Fig. 23. Argument pattern for safety requirements of Class 7.
for reasoning about such temporal properties as liveness properties (existence, progress and persistence). The main drawback of this approach is that, even though it does not require extensions of the proving support of the Rodin platform, it necessitates extension of the Event-B language by special clauses (annotations) corresponding to different types of temporal properties. Alternatively, in the cases when a temporal property can be expressed as a condition on the system control flow, the usecase/flow approach [32] described in Section 4.4 can be used.

To exemplify the instantiation of the argument pattern for safety requirements of Class 7, we consider a distributed monitoring system – Temperature Monitoring System (TMS). The full system formal specification in Event-B is presented in [26]. In brief, the TMS consists of three data processing units (DPUs) connected to operator displays in the control room. At each cycle, the system performs readings of the temperature sensors, distributes pre-processed data among DPUs where they are analysed (processed), and finally displays the output to the operator. The system model is developed in such a way that it allows for ensuring integrity of the temperature data as well as its freshness.

A safety requirement about a temporal property of this system, which we consider here, is as follows:

**SR-cl7**: Each cycle the system shall display fresh and correct data.

We leave out of the scope of this paper the mechanism of ensuring data freshness, correctness and integrity, while focusing on the fact of displaying data at each cycle. In the given Event-B specification, a new cycle starts when the event Environment is executed. To verify that the system will eventually display the data to the operator (i.e., the corresponding event Displaying will be enabled), we formulate an LTL formula for the abstract model of the system (temp_pr). Then, the instance of the mapping function $F_M$ is defined as

**SR-cl7** $\rightarrow$ \{temp_pr\}.

where

$\text{temp_pr} : = \Box (\text{after} (\text{Environment}) \rightarrow \Diamond \text{before} (\text{Displaying})).$

Here $\Box$ is an operator “always” and $\Diamond$ stands for “eventually”.

The formula $\text{temp_pr}$ has the following representation in ProB:

$$G \{ (\forall \text{main phase} \cdot \text{temp sensor} \cdot \text{curr time} \cdot \text{main phase} \in \text{MAIN PHASES} \land \text{temp sensor} \in \mathbb{N} \land \text{curr time} \in \mathbb{N} \land (\exists \text{main phase}, \text{sync t} \cdot \text{main phase} \in \text{MAIN PHASES} \land \text{sync t} \in \mathbb{N} \land \text{main phase} = \text{PROC} \land \text{temp sensor} \in \mathbb{N} \land \text{curr time} = \text{sync t}) \}$$

$$F \{ \exists \text{ss}, \text{TEMP SET} \cdot \text{main phase} = \text{DISP} \land \text{packet sent flag} = \text{TRUE} \land \text{TEMP SET} \subseteq \mathbb{N} \land \text{time progressed} = \text{TRUE} \land \text{ss} = \{x \rightarrow y \mid x, x \in \text{dom(temperature)} \land x = \text{timestamp}(i) \land y = \text{temperature}(i) \mid \text{curr time} = \text{Fresh Delta - curr time} \land (\text{ss} \neq \emptyset \rightarrow \text{TEMP SET} = \text{ss}) \land (\text{ss} = \emptyset \rightarrow \text{TEMP SET} = \text{ERR VAL})\} \}.$$

where $G$ stands for the temporal operator “globally” and $F$ is the temporal operator “finally”. These operators correspond to the standard LTL constructs “always” and “eventually” respectively. For the detailed explanation of the used variables, constants and language constructs, see [26].

In this case, the result of the model checking of this property in ProB is “no counter-example has been found, all nodes have been visited” Fig. 24 illustrates the corresponding instance of the argument pattern.

4.9. Argument pattern for SRs about timing properties (Class 8)

Another class of safety requirements that requires to be treated in a different way is Class 8 containing timing properties of the considered system. As we have already mentioned, the representation of timing properties in Event-B has not been explicitly defined yet. Nonetheless, the majority of safety-critical systems rely on timing constraints for critical functions. Obviously, the preservation of such requirements must be verified. To address this, we propose to bridge Event-B modelling with model checking of timing properties in Uppaal.

Fig. 25 shows our argument pattern for the safety requirements about timing properties. In our pattern, property, stands for some timing property to be verified, for $j \in 1, \ldots., N$, where $N$ is the number of timing properties.

Following the approach proposed by Iliassov et al. [27], we rely on the Uppaal toolset for obtaining model checking results that further can be used as the evidence in a safety case. The timing property in question can be formulated using the TCTL language. A timed automata model (an input model of Uppaal) is obtained from a process-based view extracted from an Event-B model as well as additionally introduced clocks and timing constraints. The generic mapping function $F_M$ for this class is then of the form $\text{Requirement} \rightarrow \{\text{TCTL formula}\}$.

Uppaal uses a subset of TCTL to specify properties to be checked [56]. The results of the property verification can be of three types: (1) a trace is found, i.e., a property is satisfied (user can then import the trace into the simulator); (2) a property is violated; and (3) the verification is inconclusive with the approximation used.

We propose the following steps in order to instantiate this pattern:

- a requirement Requirement should be replaced with a particular safety requirement;
- a formal development that consists of a chain of refinements in Event-B and the corresponding Uppaal model should be referred to in GSN model elements;
- the concrete mapping between the requirement and the corresponding TCTL formula should be provided, while each timing property property, from this mapping should be referred to in a GSN context element;
- model checking results on an instantiated property that have been generated by Uppaal should be included as the evidence that this property is satisfied.

We adopt a case study considered in [27,28] in order to show the instantiation of the proposed argument pattern for a safety requirement of Class 8. The case study is the Data Processing Unit (DPU) – a module of Mercury Planetary Orbiter of the BepiColombo Mission. The DPU consists of the core software and software of two scientific instruments. The core software communicates with the BepiColombo spacecraft via interfaces, which are used to receive telecommands (TCs) from the spacecraft and transmit science and housekeeping telemetry data (TMs) back to it. In this paper, we omit showing the detailed specification of the DPU in Event-B as well as we do not explain how the corresponding Uppaal model was obtained. We rather illustrate how the verified liveness and time-bounded reachability properties of the system can be reflected in the resulting safety case (Fig. 26).

The DPU is required to eventually return a TM for any received TC and must respond within a predefined time bound:

**SR-cl8**: The DPU shall eventually return a TM for any received TC and shall respond no later than the maximal response time.

We consider two timing properties associated with this requirement, i.e.,
time_pr_ex1: (new_tc == id) \rightarrow (last_tm == id).
time_pr_ex2: A[(last_tm == id && Obs1 stop) imply (Obs1_c < upper_bound)].

such that the concrete instance of the generic mapping function \( F_M \) is as follows:

\[ \text{SR-cl8} \rightarrow \{\text{time_pr_ex1, time_pr_ex2}\}. \]

The symbol \( \rightarrow \) stands for the TCTL “leads-to” operator, and id is some TC identification number. \( A[] \) stands for “Always, for any execution path” and Obs1 is a special observer process that starts the clock Obs1_c, whenever a TC command with id is received, and stops it, once the corresponding TM is returned. The variable upper_bound corresponds to the maximal response time. The corresponding instance of the argument pattern is given in Fig. 26.

5. Evaluation of the results

5.1. Discussion on the methodology

To facilitate the rigorous construction of safety cases, we have defined a set of argument patterns graphically represented using GSN. The argumentation and goal decomposition in these patterns have been influenced by the formal reasoning in Event-B.

However, since the system development based on formal methods typically requires additional reasoning about model correctness and well-definedness, we firstly introduced an argument pattern for demonstrating well-definedness of system models in Event-B. Secondly, we proposed a number of argument patterns for system safety requirements of a system. We associated these argument patterns with the classification of safety requirements presented in Section 3.2. As a result, we distinguished eight classification-based argument patterns. Despite the fact that the proposed classification of safety requirements covers a wide range of
different system safety requirements, the classification and subsequently the set of argument patterns can be further extended if needed.

Unfortunately, at the meantime not all the introduced classes of safety requirements can be formally demonstrated utilising Event-B solely. Therefore, among the proposed argument patterns there are several patterns where the evidence was constructed using the accompanying toolsets – the UseCase/Flow and ProB plug-ins for the Rodin platform, as well as the external model checker for verification of real-time systems Uppaal.

In Section 4, we exemplified the instantiation of the proposed argument patterns for safety requirements on several case studies. Among them are the sluice gate control system [24], the attitude and orbit control system [25], the system for controlling cars on a bridge [16], the temperature monitoring system [26], and the data processing unit of Mercury planetary orbiter of the BepiColombo mission [27,28].

Moreover, we have validated the proposed methodology by a larger case study – a steam boiler control system. In [29], we give a detailed description of this system, overview its formal development, as well as apply a number of the proposed argument patterns to construct the corresponding fragments of the system safety case.

Despite the fact that the accomplished Event-B development of the steam boiler control system is quite complicated and, as a result, a significant number of proof obligations have been discharged, we have not been able to instantiate two argument patterns, namely the patterns for Class 4 and Class 8. First of all, the steam boiler control system is a failsafe system, which means that there is a deadlock in its execution. Consequently, there are no requirements about the absence of system deadlock (Class 4). Second, timing properties (Class 8) were not a part of the given system requirements either. Nevertheless, the presented guidelines on instantiation of the argument patterns have allowed us to easily construct the corresponding fragments of the system safety case for the remaining safety requirements as well as to demonstrate well-definedness of the overall development of the system.

The use of the Rodin platform and the accompanying plug-ins has facilitated derivation of the proof- and model checking-based evidence that the given system safety requirements hold for the modelled system. The proof-based semantics of Event-B (a strong relationship between model elements and the associated proof obligations) has given us a direct access to the corresponding proof obligations. It has allowed us to not just claim that the verified theorems were proved but also explicitly include the obtained proof obligations into the resulting safety case.

Instantiation of the proposed argument patterns is a quite simple task. Nonetheless, the application of the overall approach requires basic knowledge of principles of safety case construction as well as a certain level of expertise in formal modelling. Therefore, experience in formal modelling and verification using state-based formalisms would be beneficial for safety and software engineers.

Currently, the proposed approach is restricted by the lack of the tool support. Indeed, manual construction of safety cases especially of large-scale safety–critical systems may be error-prone. We believe that the well-defined steps of evidence construction and the detailed guidelines on pattern instantiation given in this paper will contribute to development of the dedicated plug-in for the Rodin platform supporting the described approach.

5.2. Validation

Our approach has followed the action research paradigm [57]. Therefore, we have firstly identified a problem, namely, the need of a methodology for construction of safety cases from formal models in Event-B. Secondly, to solve this problem, we have proposed a classification of safety requirements and a set of argument patterns, which have been defined after several iterations. We validate the proposed methodology by answering the following research questions:

- Is our classification representative enough to cover safety–critical control systems?
- Are the fragments of a safety case constructed according to our methodology internally complete (i.e., is there any claim (goal) in the argument such that a path from it does not end in evidence (solution))?
- Is the described process within the proposed approach to construction of safety cases comprehensible?
• Is the proposed approach sufficiently scalable to handle large-scale industrial systems, i.e., does it not introduce prohibitive complexity?

To answer these research questions, we performed experiments on six representative case studies from different domains. The majority of the chosen case studies has been developed within European projects such as RODIN [17] and DEPLOY [18] together with the involved industrial partners. Nevertheless, neither of the case studies can be used to illustrate instantiation of all the proposed argument patterns. We demonstrate which case study is used for which argument pattern in Table 2. In Table 2 and further tables, WD corresponds to the argument pattern for well-definedness of formal development, while 1–8 stand for the argument patterns for classes 1–8 respectively. All together, the chosen case studies cover all the classes of safety requirements proposed in this paper.

The internal completeness of safety cases can be measured by a metric called the coverage of claims (goals) [58]. Tables 3 and 4 show the coverage of claims (COV) calculated as the proportion of the total number of developed claims (CP) to the total number of claims (C). Specifically,

\[
COV = \frac{CP}{C}
\]

However, the provided calculations cover only the constructed fragments of safety cases illustrating the argument patterns instantiation. Building complete safety cases is still needed for more extensive evaluation of the approach.

To summarise, firstly, our classification has shown to be expressive enough for formalising in Event-B and accompanying tools the selected safety–critical systems from different domains. Secondly, the fragments of a safety case obtained by instantiating the proposed patterns are internally complete. We have shown this by calculating a specific metric and demonstrating the results in Tables 3 and 4. Thirdly, to evaluate whether the proposed process is comprehensible, we have performed internal experiments in the group and repeatedly refined such aspects of the approach as the classification of safety requirements, the resulting argument structures for safety cases, and the choice of case studies. Moreover, both requirement classification and proposed rigorous mappings between requirements and the corresponding model elements have been strongly influenced by cooperation with our industrial partners, thus giving us a necessary reality check. Finally, despite the fact that a system safety case inevitably becomes larger and more complex for systems with a higher level of complexity, our approach does not prohibitively increase the resulting complexity of the constructed safety case. That is due to orthogonality of the classification of safety requirements on which it is based. Within the classification, each class is associated with an independent procedure for formalising a safety requirement of this class and constructing a fragment of the safety case demonstrating that this requirement has been met. This allows us to avoid the exponential growth of added complexity while constructing safety cases of safety–critical systems.

5.3. Threats to validity

In this section, we discuss internal and external threats to the validity of the presented methodology.

Internal validity: Researcher bias is a key threat to the internal validity of our approach. Specifically, we tend to rely on the Event-B formalism too much. Event-B has evolved from Action Systems [59,60], Refinement Calculus [61] and the B Method [30]. These formalisms are well-defined and mathematically-grounded methods that we are experts in. However, as any method, Event-B has its limitations. To overcome these limitations, we plan to integrate Event-B with the existing tools via the Rodin platform. There are already a number of Rodin plug-ins allowing for modelling language extensions, animation and model checking, documentation, SMT solvers, code generation, bridging with other formalisms, etc. [62].

In this paper, we have not taken into account all the quantitative aspects of safety. Specifically, we consider only timing properties, while leaving aside, e.g., probabilistic system characteristics such as failure rates. To mitigate this threat, our approach can be extended with probabilistic reasoning for representing and verifying requirements involving quantitative characteristics or constraints. This can be achieved via bridging our approach with the existing works that aim at implementing quantitative reasoning in Event-B, e.g., [63,64].

External validity: The authors of this paper are not industrial practitioners. Nevertheless, we have been involved in several big European and national projects, involving both academic and industrial partners. As a result, we have been able to work on realistic case studies and discuss them with the industrial practitioners. Some of these case studies have been used as illustrative examples in this paper. Our experience from previous projects and cooperation with industrial partners has led us to a representative classification of safety requirements proposed in this paper. However, as the whole, the proposed approach has not been applied in industry yet. To mitigate this threat, we plan to obtain the feedback on our work from our industrial partners.

Unfortunately, the current lack of a tool support for our approach makes the construction of a complete safety case for a complex safety–critical system very labour- and time-consuming. This precludes us from fully analysing the resulting safety cases based on the derived statistical information (e.g., the number of nodes and claims for a particular class, etc.). Obtaining adequate tool support for the proposed approach is our current priority. This would also facilitate conducting of additional case studies that are needed to confirm both applicability and scalability of the proposed approach.

We believe that the results of our research can be generalised. First, our approach can be extended without modifying the already proposed methodology (with new requirements classes, external tools for verification, etc). Second, we have successfully applied

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<th>Case study</th>
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<td>WD</td>
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<td>Steam boiler control system</td>
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<td>Sluice gate control system</td>
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<td>System for controlling cars on a bridge</td>
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<td>Temperature monitoring system</td>
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the proposed approach to a number of case studies from different domains. Finally, even though we have used GSN to represent the argument patterns in a structured way, the demonstrated argumentation based on formal reasoning is independent of the notation and therefore can be represented in another formats, e.g., textually.

6. Related work

In this section, we overview related contributions according to the following three directions: firstly, we consider the publications on the use of formal methods for safety cases; secondly, we overview the works that aim at formalising safety requirements; and thirdly, we take a closer look at the approaches focusing on argument patterns.

**Formal methods in safety cases.** There are two main research directions in utilising formal methods for safety cases. On the one hand, a safety case argument itself can be formally defined and verified. On the other hand, safety requirements can be formalised and formally verified allowing us to derive the safety evidence such as the obtained results of static analysis, model checking or theorem proving. Note that such evidence corresponds to the class of safety evidence called *formal verification results* defined in the safety evidence taxonomy proposed by Nair et al. in [7].

In the former case, soundness of a safety argument can be proved by means of theorem proving in the classical or higher order logic, e.g., using the interactive theorem prover PVS [65,66]. In particular, Rushby [66] formalises a top-level safety argument to support automated checking of soundness of a safety argument. He proposes to represent a safety case argument in the classical logic as a theorem where antecedents are the assumptions under which a system (or design) satisfies the consequent, whereas the consequent is a specific claim in the safety case that has to be assured. Then, such a theorem can be verified by an automated interactive theorem prover or a model checker.

In the latter case, soundness of an overall safety case is not formally examined. The focus is rather put on the evidence derived from formal analysis to show that the specific goals reflecting safety requirements are met. For example, to support the claim that the source code of a program module does not contain potentially hazardous errors, the authors of [67] use as the evidence the results of static analysis of program code. In [11,13], the authors assure safety of automatically generated code by providing formal proofs as the evidence. They ensure that safety requirements hold in specific locations of software system implementations. In [58,68], the authors automate generation of heterogeneous safety cases including a manually developed top-level system safety case, and lower-level fragments automatically generated from the formal verification of safety requirements. According to this approach, the implementation is formally verified against a mathematical specification within a logical domain theory. This approach is developed for the aviation domain and illustrated by an unmanned aircraft system. To ensure that a model derived during model-driven development of a safety critical system, namely pacemaker, satisfies all the required properties, the authors of [14] use the obtained model checking results. Our approach proposed in this paper also belongs to this category. Formalisation and verification of safety requirements of a critical system allows us to obtain the proof- and model checking-based evidence that these requirements hold.

Moreover, recent developments in the area of safety cases include introduction of querying GSN safety case argument structures, which are used to provide argument structure views [69]. Specifically, GSN nodes are semantically enriched by metadata, given as a set of attributes. Then, the Argument Query Language (AQL) is used to query safety case argument structures and create views. The methodology proposed in [69] can be useful to address stakeholders’ concerns about system safety, and can facilitate allocating and displaying the information of interest. We consider this work as complementary to our approach.

**Formalisation of safety requirements.** Incorporation of requirements in general, and safety requirements in particular, into formal specifications is considered to be a challenging task. We overview some recent approaches that address this problem dividing them into two categories: those that aim at utilising model checking for verification of critical properties, and those that employ theorem proving for this purpose.
For example, a formalisation of safety requirements using the Computation Tree Logic (CTL) and then verification of them using a model checker is presented in [70]. The author classifies the given requirements associating them with the corresponding CTL formulas. A similar approach is presented in [71]. Here safety properties defined as LTL formulas are verified by using the SPIN model checker.

In contrast, the authors of [72] perform a systematic transformation of a Petri net model into an abstract B model for verification of safety properties by theorem proving. Another work that aims at verifying safety requirements by means of theorem proving is presented in [24]. The authors incorporate the given requirements into an Event-B model via applying a set of automated patterns, which are based on Failure Modes and Effects Analysis (FMEA).

Similarly to these works, we take an advantage of using theorem proving and a refinement-based approach to formal development of a system. We gradually introduce the required safety properties into an Event-B model and verify them in the refinement process. This allows us to avoid the state explosion problem commonly associated with model checking, thus making our approach more scalable for systems with higher levels of complexity. Nonetheless, in this paper, we also rely on model checking for those properties that cannot be verified by our framework directly.

Furthermore, there are other works that aim at formalising safety requirements, specifically in Event-B [46–48,73]. Some of them propose to incorporate safety requirements as invariants and before-after predicates of events [47,48], while others, e.g., [46], represent them as invariants or theorems only. Moreover, all these works show the correspondence between some particular requirements and the associated elements of the Event-B structure. However, they neither classify the safety requirements nor give precise guidelines for formal verification of those requirements that cannot be directly verified by the Event-B framework. In contrast, to be able to argue over each given safety requirement by relying on its formal representation, we propose a classification of safety requirements and define a precise mapping of each class onto a set of the corresponding model expressions. Moreover, for some of these classes, we propose bridging Event-B with other tools (model checkers).

Argument patterns. In general, argument patterns (or safety case patterns) facilitate construction of a safety case by capturing commonly used structures and allowing for simplified instantiation. Safety case patterns have been introduced by Kelly and McDermid [6] and received recognition among safety case developers. In [74], the authors give a formal definition for safety case patterns, define formal semantics of patterns, and propose a generic data model and algorithm for pattern instantiation. For example, a safety case pattern for arguing the correctness of implementations developed from a timed automata model using a model-based development approach has been presented in [39]. An instantiation of this pattern has been illustrated on the implementation software of a patient controlled analgesic infusion pump. In [37], the author proposes a set of property-oriented and requirement-oriented safety case patterns for arguing safe execution of automatically generated program code with respect to the given safety properties as well as safety requirements. Additionally, the author defines architecture-oriented patterns for safety-critical systems developed using a model-based development approach.

An approach to automatically integrating the output generated by a formal method or tool into a software safety assurance case as an evidence argument is described in [15]. To capture the reasoning underlying a tool-based formal verification, the authors propose specific safety case patterns. The approach is software-oriented. A formalised requirement is verified to hold at a specific location of code. The proposed patterns allow formal reasoning and evidence to be integrated into the language of assurance arguments. Our approach is similar to the approach presented in [15]. However, we focus on formal system models rather than the code. Moreover, the way system safety requirements are formalised and verified in Event-B varies according to the proposed classification of safety requirements. Consequently, the resulting evidence arguments are also different. Nevertheless, we believe that the approach given in [15] can be used to complement our approach.

In this paper, we contribute to a set of existing safety case patterns and describe in detail their instantiation process for different classes of safety requirements. Moreover, our proposed patterns facilitate construction of safety cases where safety requirements are verified formally and the corresponding formal-based evidence is derived to represent justification of safety assurance. The evidence arguments obtained by applying our approach explicitly reflect the formal reasoning instead of just references to the corresponding proofs or the model checking results.

7. Conclusions

In this paper, we propose a methodology supporting rigorous construction of safety cases from formal Event-B models. It guides the developers starting from informal representation of system safety requirements to building the corresponding parts of safety cases via formal modelling and verification in Event-B and the accompanying toolsets.

We believe that the proposed methodology has shown good scalability. In this paper, we have illustrated the application of our methodology by a number of small examples, while the reader can find demonstration of its application to a larger case study, specifically, a steam boiler control system, in our technical report [29]. We have observed that, independently of the system complexity, the proposed mechanism of mapping safety requirements into formal models involves a limited number of model elements as well as the model theorems to be verified and thus requires a limited amount of proving effort. Moreover, the introduced classes of safety requirements and the associated argument patterns can be mostly used independently from each other. Therefore, based on the conducted case studies as well as these observations, we believe that the growth of the added complexity associated with application of our approach is non-exponential.

The accomplished formal development has confirmed our observation that to construct an adequate safety case of a system based on its formal model, a feedback loop between two processes, namely, the process of formal system development and construction of safety cases, is required. It means that, if construction of a safety case indicates that the associated formal model is “weak”, i.e., it does not contain an adequate formalisation of some safety requirements that need to be demonstrated in the safety case, the developers should be able to react on that by improving the model.

Our main contribution, namely, the proposed methodology for rigorous construction of safety cases, has led us to achieving the following two sub-contributions. Firstly, we have classified safety requirements and shown how they can be formalised in Event-B. To attain this, we have proposed a strict mapping between the given safety requirements and the associated elements of formal models, thus establishing a clear traceability of those requirements. Secondly, we have proposed a set of argument patterns based on the proposed classification, the use of which facilitates the construction of safety cases. Due to the strong relationship between model elements and the associated proof obligations provided by the proof-based semantics of Event-B, we have been able to formally verify the mapped safety requirements and derive the
corresponding proofs. Moreover, via developing the argument strategies based on formal reasoning and using the resulting proofs as the evidence for a safety case, we have achieved additional assurance that the desired safety requirements hold.

Furthermore, application of the well-defined Event-B theory for formal verification of safety requirements and formal-based construction of safety cases has led to the use of particular safety case goals or strategies. It has allowed us to omit the additional explanations why the defined strategies are needed and why the proposed evidence is relevant. Otherwise, we would have needed to extend each proposed argument pattern with multiple instances of a specific GSN element called justification [38]. Consequently, this would have led to a significant growth of already large safety cases.

In this work, we have focused on system safety aspects, however, the proposed approach can be extended to cover other dependability attributes, e.g., reliability and availability. The theoretical basis for such extensions has been already developed in [63,64]. We also believe that the generic principles described in this paper for the Event-B formalism are applicable to any other formalism defined as a state transition system, e.g., B, Z, VDM, refinement calculus, etc.

So far, all the proposed patterns and their instantiation examples have been developed manually. However, the larger a system under consideration is, the more difficult this procedure becomes. Therefore, the necessity of automated tool support is obvious. We consider development of a dedicated plug-in for the Rodin platform as a part of our future work. Moreover, the proposed classification of the safety requirements is by no means complete. Consequently, it could be further extended with some new classes and the corresponding argument patterns.

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